2014 Korea International Mathematics Competition

21~26 July, 2014, Daejeon City, Korea

Invitational World Youth Mathematics Intercity Competition

TEAM CONTEST

Time：60 minutes

Instructions:

- Do not turn to the first page until you are told to do so.
- Remember to write down your team name in the space indicated on every page.
- There are 10 problems in the Team Contest, arranged in increasing order of difficulty. Each question is printed on a separate sheet of paper. Each problem is worth 40 points. For Problems 1, 3, 5, 7 and 9, only numerical answers are required. Partial credits will not be given. For Problems 2, 4, 6, 8 and 10, full solutions are required. Partial credits may be given.
- The four team members are allowed 10 minutes to discuss and distribute the first 8 problems among themselves. Each student must attempt at least one problem. Each will then have 35 minutes to write the solutions of their allotted problem independently with no further discussion or exchange of problems. The four team members are allowed 15 minutes to solve the last 2 problems together.
- No calculator, calculating device, electronic devices or protractor are allowed.
- Answer must be in pencil or in blue or black ball point pen.
- All papers shall be collected at the end of this test.

English Version

For Juries Use Only

No.	1	2	3	4	5	6	7	8	9	10	Total	Sign by Jury
Score												
Score												

TEAM CONTEST

23rd July, 2014, Daejeon City, Korea

Team: ____________________ Score: ______________

1. Let $S_1 - S_2 + S_3 - S_4 + S_5 = \dfrac{m}{n}$ where *m* and *n* are relatively prime positive integers and

$$S_1 = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6},$$

$$S_2 = \frac{1}{2\times 3} + \frac{1}{2\times 4} + \frac{1}{2\times 5} + \frac{1}{2\times 6} + \frac{1}{3\times 4} + \frac{1}{3\times 5} + \frac{1}{3\times 6} + \frac{1}{4\times 5} + \frac{1}{4\times 6} + \frac{1}{5\times 6},$$

$$S_3 = \frac{1}{2\times 3\times 4} + \frac{1}{2\times 3\times 5} + \frac{1}{2\times 3\times 6} + \frac{1}{2\times 4\times 5} + \frac{1}{2\times 4\times 6} + \frac{1}{2\times 5\times 6} + \frac{1}{3\times 4\times 5} + \frac{1}{3\times 4\times 6} + \frac{1}{3\times 5\times 6} + \frac{1}{4\times 5\times 6},$$

$$S_4 = \frac{1}{2\times 3\times 4\times 5} + \frac{1}{2\times 3\times 4\times 6} + \frac{1}{2\times 3\times 5\times 6} + \frac{1}{2\times 4\times 5\times 6} + \frac{1}{3\times 4\times 5\times 6},$$

$$S_5 = \frac{1}{2\times 3\times 4\times 5\times 6}.$$

Determine the value of *m* + *n*.

Answer: ______________________________

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2. The distinct prime numbers p, q, r and s are such that $p+q+r+s$ is also a prime number, and both p^2+qr and p^2+qs are squares of integers. Determine $p+q+r+s$.

Answer: ________________________

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3. Determine the sum of all integers *n* for which $9n^2+23n-2$ is the product of two positive even integers differing by 2.

Answer: ______________________________

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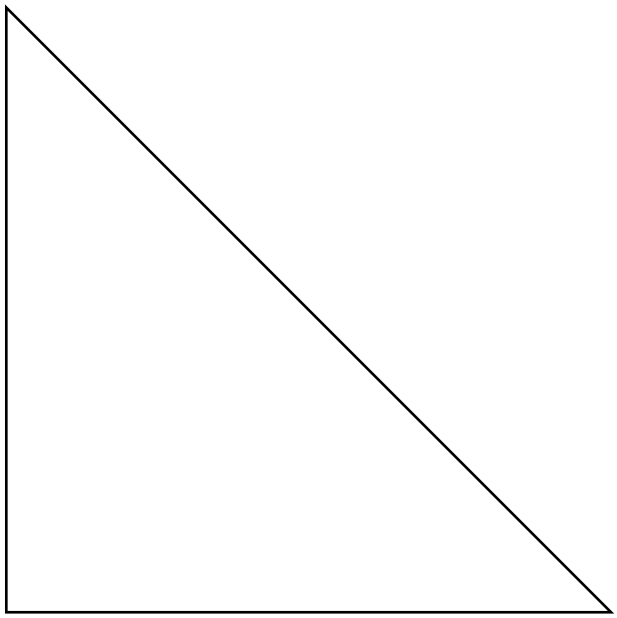
4. Cut a right isosceles triangle into the minimum number of pieces which may be assembled to form, without gaps or overlaps, two right isosceles triangles of different sizes.



Answer: ____

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5. Determine the maximum value of $a + b + c$ where a, b and c are positive integers such that $2b + 1$ is divisible by a, $2c + 1$ is divisible by b and $2a + 1$ is divisible by c.

Answer: ______________________________________

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Team： ______________________ Score： ______________

6. Determine all positive integers under 100 with exactly four positive divisors such that the difference between the sum of two of them and the sum of the other two is the square of an integer.

Answer: ______________________________

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7. A Korean restaurant offers one kind of soup each day. It is one of fish soup, beef soup or ginseng chicken soup, but it will not offer ginseng chicken soup three days in a row. Determine the number of different seven-day menus.

Answer: ______________________________

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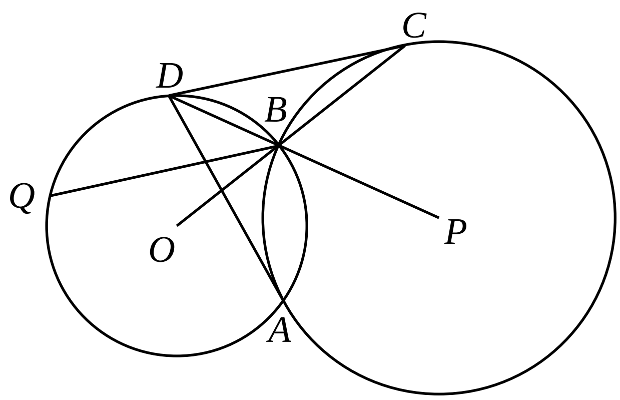
Team：________________________ Score：______________

8. Two circles, with centres O and P respectively, intersect at A and B. The extension of OB intersects the second circle at C and the extension of PB intersects the first circle at D. A line through B parallel to CD intersects the first circle at $Q \neq B$. Prove that $AD = BQ$.



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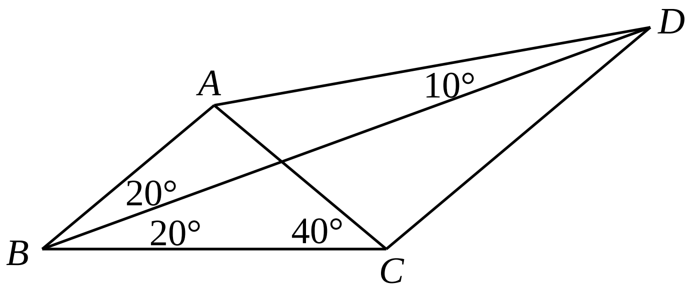
9. In the quadrilateral $ABCD$, $\angle BDA = 10°$, $\angle ABD = \angle DBC = 20°$ and $\angle BCA = 40°$. Determine the measure, in degrees, of $\angle BDC$.



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Answer: __________________________________

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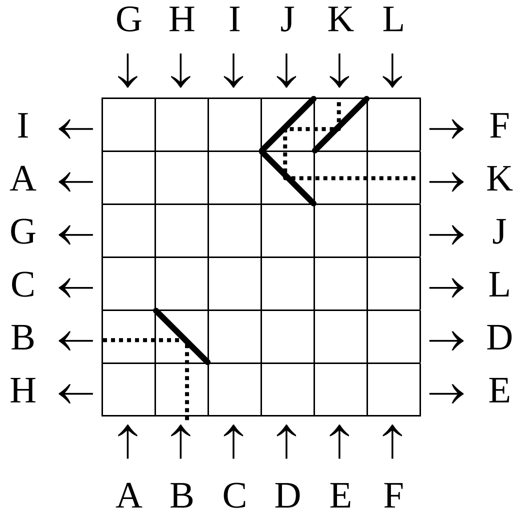
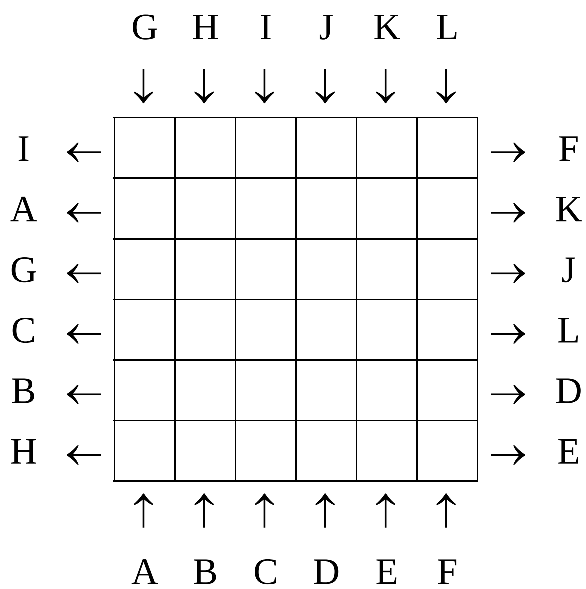
10. The diagram below shows a 6×6 box. Balls A to L enter along columns and exit along rows. The point of entry and the point of exit of each ball are marked by its own letter. Reflectors may be placed along either diagonal of any square in the box, four of which are shown as an illustration. When a ball hits a reflector, it bounces off in a perpendicular direction. You must make every ball go to the right place, as illustrated by the balls B and K. You must remove the four reflectors used in the illustration, and then place ten reflectors in other squares. You may not put any reflector in the same position as in the illustration.





Answer: _______________