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BULGARIA INTERNATIONAL MATHEMATICS COMPETITION

BURGAS • 01.07 - 06.07 • 2018

Invitational World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes

Information:

- You are allowed 120 minutes for this paper, consisting of 12 questions in Section A to which only numerical answers are required, and 3 questions in Section B to which full solutions are required.
- Each question in Section A is worth 5 points. No partial credits are given. There are no penalties for incorrect answers, but you must not give more than the number of answers being asked for. For questions asking for several answers, full credit will only be given if all correct answers are found. Each question in Section B is worth 20 points. Partial credits may be awarded.
- Diagrams shown may not be drawn to scale.

Instructions:

- Write down your name, your contestant number and your team's name in the space provided on the first page of the question paper.
- For Section A, enter your answers in the space provided after the individual questions on the question paper. For Section B, write down your solutions on spaces provided after individual questions.
- You must use either a pencil or a ball-point pen which is either black or blue.
- You may not use instruments such as protractors, calculators and electronic devices.
- At the end of the contest, you must hand in the envelope containing the question paper and all scratch papers.

English Version

Team:______________ ***Name***:__________________ ***No.***: _____________ ***Score:***__________

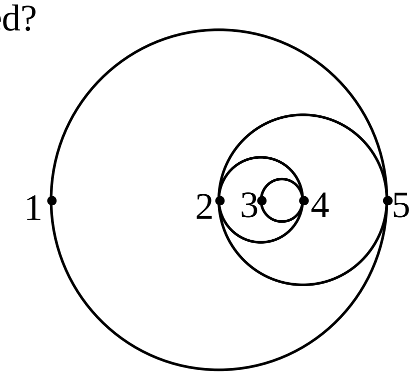
For Juries Use Only

No.	Section A												Section B			Total	Sign by Jury
	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3		
Score																	
Score																	

Section A.

In this section, there are 12 questions. Fill in the correct answer in the space provided at the end of each question. Each correct answer is worth 5 points.

1. The diagram shows five collinear towns connected by semicircular roads. A journey is defined as travelling between two towns along a semicircle. In how many possible ways can we start and end at Town 5 after four journeys if the journeys may be repeated?



Answer：________ways

2. Let m and n be positive integers such that $m(n-m)=-11n+8$. Find the sum of all possible values of $m-n$.

Answer：____________

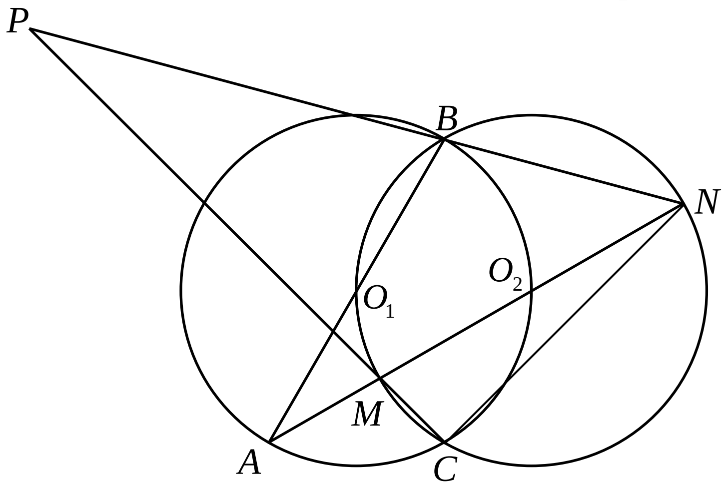
3. Ann tosses a fair coin twice while Bob tosses the same coin three times. The probability that they obtain the same number of heads at the end of the game is expressed as an irreducible fraction. What is the sum of its numerator and its denominator?

Answer：_____________

4. Let p and q be prime numbers such that $p^2+3pq+q^2$ is the square of an integer. Find the largest possible value of $p+q$.

Answer：_____________

5. Two circles k_1 and k_2 with the same radius intersect at points B and C. The center O_1 of k_1 lies on k_2 and the center O_2 of k_2 lies on k_1. AB is a diameter of k_1, and AO_2 intersects k_2 at points M and N, with M between A and O_2. The extensions of CM and NB intersect at point P. Find CP : CN.



Answer：_____:______

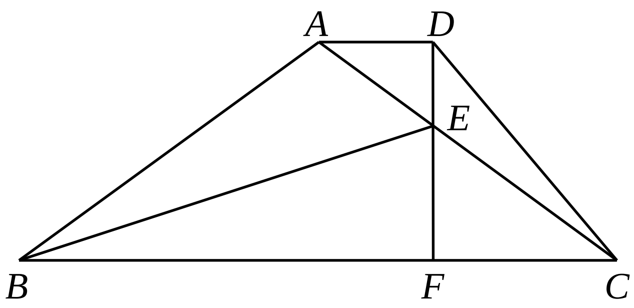
6. Given is the product 1!2!3!..... 99!100! How many consecutive 0s are there at the end of this product?

Answer：_____________

7. Let $P(x)=x^4+ax^3+bx^2+cx+d$, where *a*, *b*, *c* and *d* are real constants. Suppose $P(1)=7$, $P(2)=52$ and $P(3)=97$. Find the value of $\dfrac{P(9)+P(-5)}{4}$.

Answer：_____________

8. In quadrilateral *ABCD*, *AD* is parallel to *BC* and $AB = AC$. *F* is a point on *BC* such that *DF* is perpendicular to *BC*. *AC* intersects *DF* at *E*. If $BE = 2DF$ and *BE* bisects $\angle ABC$, find the measure, in degrees, of $\angle BAD$.



Answer：_____________ ○

9. Arrange the numbers 1, 2, 3, 4, 5, 6 and 7 in a row such that none of the first number, the sum of the first two numbers, the sum of the first three numbers, and so on, up to the sum of all seven numbers, is divisible by 3. In how many ways can this be done?

Answer：________ ways

10. An equilateral triangle and a regular 7-sided polygon are inscribed in the same circle of circumference 84 cm, divided by the vertices into ten arcs. What is the maximum possible length, in cm, of the shortest arc?

Answer：___________ cm

11. If *a* and *b* are real numbers such that $\sqrt[3]{a}-\sqrt[3]{b}=12$ and $ab=\left(\dfrac{a+b+8}{6}\right)^3$, find the value of $a-b$.

Answer：_____________

12. How many ordered triples (*x*, *y*, *z*) of real numbers are there such that $x+y^2=z^3$, $x^2+y^3=z^4$ and $x^3+y^4=z^5$?

Answer：_____________

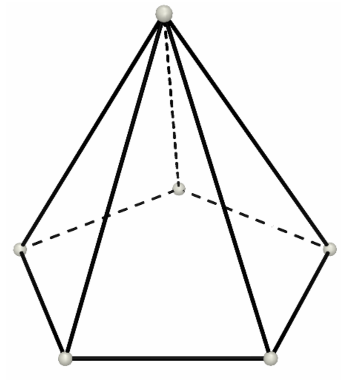
Section B.

Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. Determine the value of $a+b$ if the equation $|x^2-2ax+b|=8$ has only three real roots, which are the sides of a right triangle.

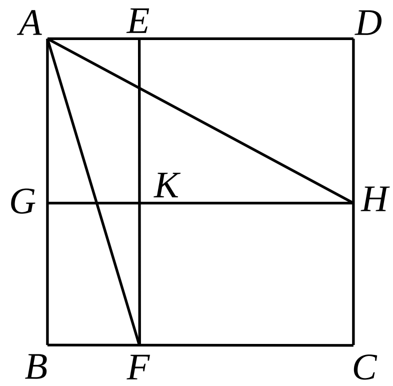
Answer : _____________

2. In how many ways can you paint the six vertices of a regular pentagonal pyramid using at most six different colours, such that two vertices connected by an edge have different colours? If the result of one way of painting may be obtained by rotation from the result of another way of painting, only one of them will be counted.



Answer：_________ ways

3. Let *ABCD* be a square. *E* and *F* are points on *AD* and *BC* respectively such that *EF*//*AB*. *G* and *H* are points on *AB* and *DC* respectively such that *GH*//*AD*. *EF* and *GH* intersect at *K*. If the area of *KFCH* is equal to twice that of *AGKE*, find the measure, in degrees, of $\angle FAH$.



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Answer : ______________