BULGARIA INTERNATIONAL MATHEMATICS COMPETITION

BIMC 2018

BURGAS • 01.07 - 06.07 • 2018

Invitational World Youth Mathematics Intercity Competition

Team Contest

Time limit: 70 minutes

Information:

- You are allowed 70 minutes for this paper, consisting of 10 questions printed on separate sheets. For questions 1, 3, 5, 7 and 9, only numerical answers are required. For questions 2, 4, 6, 8 and 10, full solutions are required.
- Each question is worth 40 points. For odd-numbered questions, no partial credits are given. There are no penalties for incorrect answers, but you must not give more than the number of answers being asked for. For questions asking for several answers, full credit will only be given if all correct answers are found. For even-numbered questions, partial credits may be awarded.
- Diagrams shown may not be drawn to scale.

Instructions:

- Write down your team's name in the space provided on every question sheet.
- Enter your answers in the space provided after the individual questions on the question paper.
- During the first 10 minutes, the four team members examine the first 8 questions together, and altogether discuss them. Then they distribute the questions among themselves, with each team member is allotted at least 1 question.
- During the next 35 minutes, the four team members write down the solutions of their allotted problems on the respective question sheets, with no further communication / discussion among themselves.
- During the last 25 minutes, the four team members work together to write down the solutions of the last 2 questions on the respective questions sheets.
- You may not use instruments such as protractors, calculators and electronic devices.
- At the end of the contest, you must hand in the envelope containing all question sheets and all scratch papers.

English Version

Team: _____ ***Score:*** _____

For Juries Use Only

No.	1	2	3	4	5	6	7	8	9	10	Total	Sign by Jury
Score												
Score												

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TEAM CONTEST

3rd July, 2018, Burgas, Bulgaria

Team：_____ Score：_____

1. How many positive integers less than 2018 can be expressed as a sum of exactly three different positive divisors of themselves?

Answer: _____ positive integers

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Team：_____ Score：_____

2. How many pairs of positive integer (x, y) are there such that $x < y$ and $\dfrac{x^2+y^2}{x+y}$ is an integer which is a divisor of 2835?

Answer: _____ pairs

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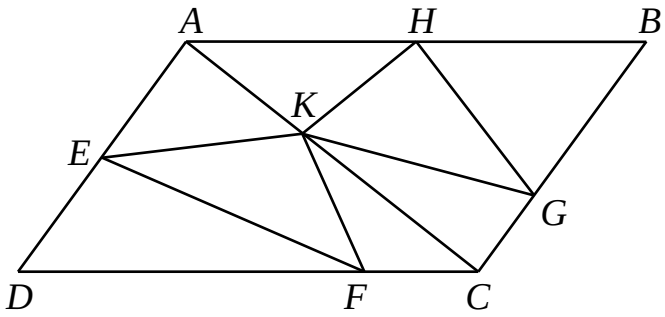
Team : _____ Score : _____

3. Let $ABCD$ be a parallelogram of area 240 cm^2. E is the midpoint of AD and H is the midpoint of AB. G is a point on BC such that $BG = 2GC$ and F is a point on CD such that $DF = 3FC$. K is a point on AC such that the area of triangle EKF is 33cm^2. Find the area, in cm^2, of triangle HKG.



Answer: _____ cm^2

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Team：_____ Score：_____

4. Find the largest positive integer m such that $m^4+16m+8$ is the product of two or more consecutive integers.

Answer: _____

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Team：_____ Score：_____

5. For which positive integer value of k does $\frac{20^k + 18^k}{k!}$ attain its maximum value?

Answer: _____

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6. The Roman army has 2018 units guarding their provinces. The Emperor was worried that when there are at least 64 units in a province, they might get together and overthrow the Emperor. So on each day, he visited one such potentially troublesome province and sent all its units to other provinces, no two going to the same province. Prove that after 64 days, there were no more provinces with at least 64 units.

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7. The sum of all positive integers that are not relatively prime to 2018 and have exactly 2017 positive divisors is divided by 2019. Find the remainder of this division.

Answer: _____

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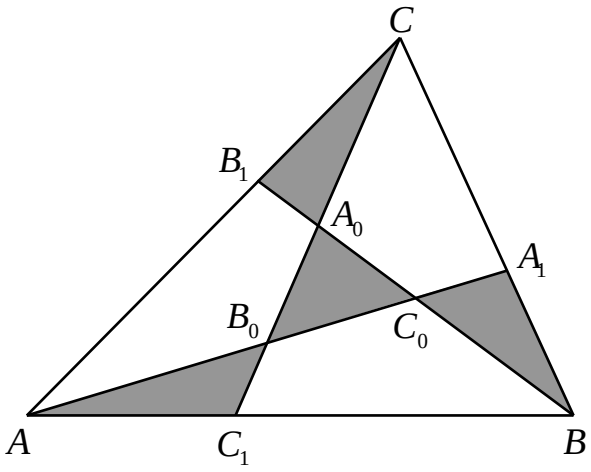
8. In the diagram below, the triangle is dissected into four smaller triangles and three quadrilaterals. Each smaller triangle has area 1cm^2. Find the area, in cm^2, of quadrilateral $CA_0C_0A_1$.



Answer: _____ cm^2

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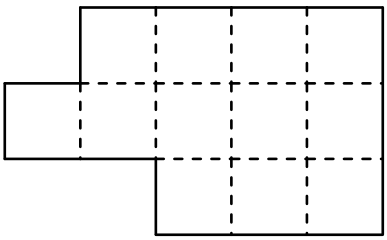
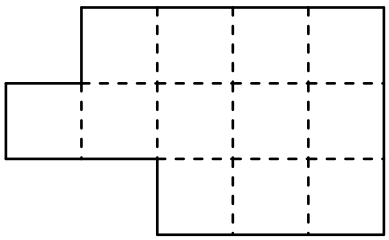
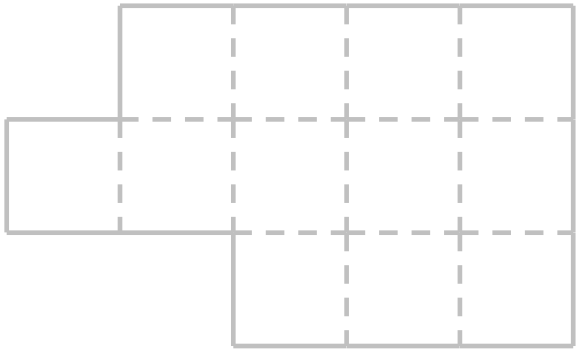
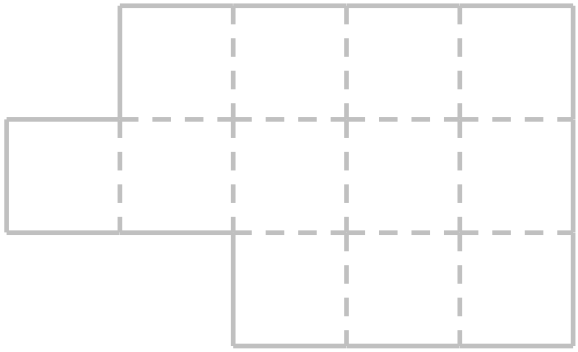
9. Dissect the first figure below, not necessarily along the grid lines, into three pieces such that the pieces obtained are identical up to rotation and reflection. Then dissect the second figure, not necessarily along the grid lines, into four pieces such that the pieces obtained are identical up to rotation and reflection.





Answer: First figure Second figure





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10. Tom writes down the number 1. For any number n already written down, Tom writes down the numbers $5n$ and $5n+1$, provided that they are less than 1000. Jerry computes all sums of two different numbers Tom has written down. How many different sums does Jerry get?

Answer: _____