SOUTH AFRICAN INTERNATIONAL MATHEMATICS COMPETITION

Durban • 1 to 6 August 2019

Elementary Mathematics International Contest

Individual Contest

Time limit: 90 minutes

Information:

- You are allowed 90 minutes for this paper, consisting of 15 questions to which only numerical answers are required.
- Each question is worth 10 points. No partial credits are given. There are no penalties for incorrect answers, but you must not give more than the number of answers being asked for. For questions asking for several answers, full credit will only be given if all correct answers are found.
- Diagrams shown may not be drawn to scale.

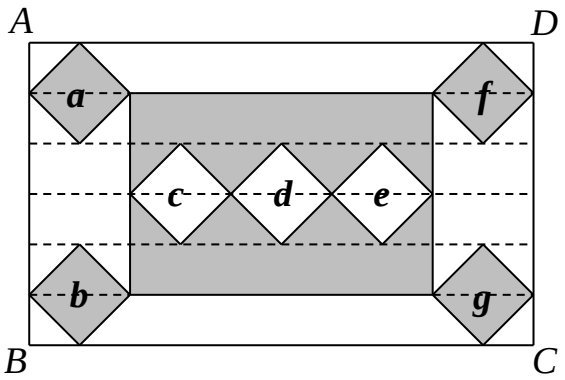
Instructions:

- Write down your name, your contestant number and your team's name on the answer sheet.
- Enter your answers in the space provided on the answer sheet.
- You must use either a pencil or a ball-point pen which is either black or blue.
- You may not use instruments such as protractors, calculators and electronic devices.
- At the end of the contest, you must hand in the envelope containing the question paper, your answer sheet and all scratch papers.

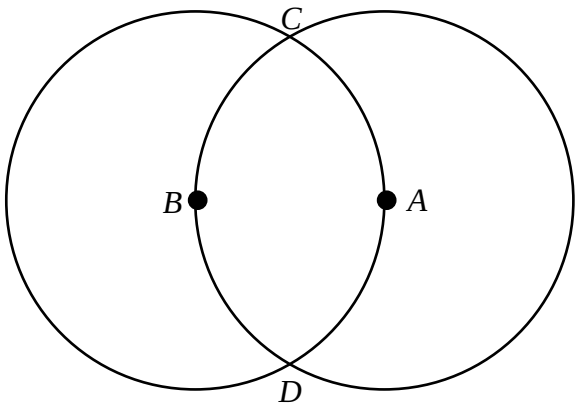
English Version

Team: _____ ***Name:*** _____ ***No.:*** _____

1. In the figure below, a, b, c, d, e, f and g are identical squares. What is the ratio of the shaded area to the area of rectangle $ABCD$?



2. In the figure below, there are two intersecting circles of the same size, with centres A and B, such that each circle's centre lies on the other circle's circumference. What is the ratio of the length of the arc CBD to the circumference of one circle?

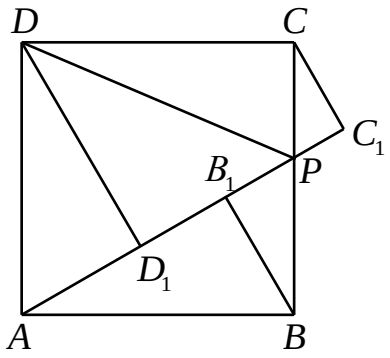


3. Each of three students A, B and C has a calculator and starts doing some operations at the same time. Student A starts from 1509 and adds 3 at each step, student B starts from 2019 and subtracts 7 at each step, while student C starts from the number X and adds 1 at the first step, adds 2 at the second step, adds 3 at the third step and so on. If after N steps the three students end up with the same result, then find the number X.

4. The city government surveyed all of the grade 5 and 6 students to determine the popularity of riding a scooter and skateboarding. Of the grade 5 students, 9% answered that they like riding a scooter and 14% answered that they like skateboarding. Of the grade 6 students, 11% answered that they like riding a scooter and 7% answered that they like skateboarding. When combining the grade 5 and 6 students, the number of students that like riding a scooter is the same as the number of students that like skateboarding. If the total number of grade 5 and 6 students in the city is 59400 and no student likes both riding a scooter and skateboarding, determine the number of grade 6 students in the city.

5. The average of three positive integers is 2019. If one of the numbers is replaced by 3, then the average of these 3 positive integers becomes 673. What is the original number replaced by 3?

6. The figure below shows a square *ABCD*, where point *P* lies on *BC* and *AP* = 4 cm. Let BB_1, CC_1 and DD_1 be the perpendiculars from the points *B*, *C* and *D* to the line *AP* respectively. If $BB_1 + CC_1 + DD_1 = 6$ cm, find the area, in cm^2, of triangle *APD*.

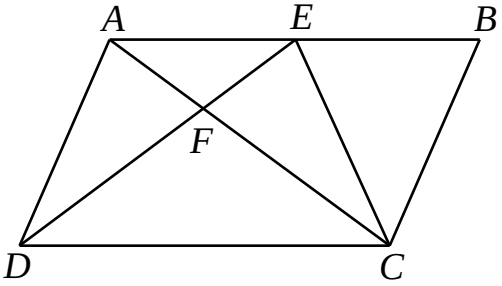


7. Nine baskets contain the following number of eggs: 4, 5, 7, 8, 12, 13, 14, 23 and 24. Anyone who wants to buy eggs must buy all the eggs in a basket. Kitty and Philip bought four baskets each. If Kitty got three times as many eggs as Philip, then how many eggs are there in the remaining basket?

8. Divide the numbers 1 through 9 into three groups of three and compute the product of the three numbers in each group such that the largest product is as small as possible. What is this largest product?

9. If *A* has more than one prime divisor, how many distinct positive divisors must *A* have such that A^2 has 2019 distinct positive divisors?

10. Let *ABCD* be a parallelogram and point *E* be the midpoint of *AB*, as shown in the figure below. If the area of the triangle *CDF* is 2692 cm^2, then find the area, in cm^2, of the triangle *BCE*.

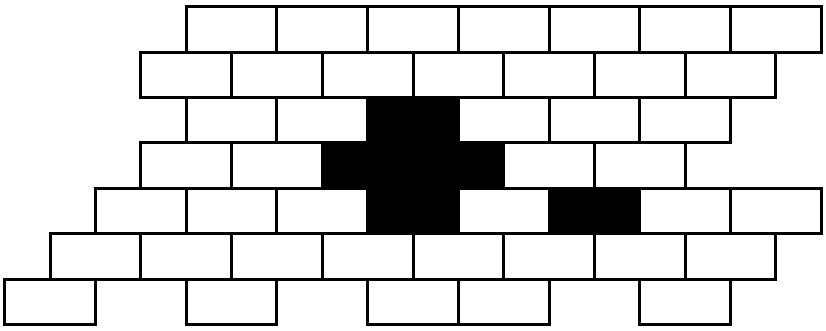


11. The side lengths of a triangle are consecutive positive integers, in cm. One of its medians is perpendicular to one of its angle bisectors. What is the perimeter, in cm, of this triangle?

12. Consider all the words obtained as permutations of the letters {D, U, R, B, A, N}. All these words are written in a row, arranged in alphabetical order, starting from ABDNRU, and ending with URNDBA. What is the position of the word DURBAN in this sequence?

13. John writes down all integers from 1 up to 2019 inclusive on the blackboard. How many even digits did he use?

14. The figure below illustrates a 7-row brick wall with 2 holes (shaded in black). We want to choose a brick in each row such that any two bricks chosen in any adjacent rows are connected (two bricks are connected if they share at least a portion of their sides). In how many different ways can we choose the 7 bricks?



15. In the following equation, each of the letters *S*, *A*, *I*, *M*, *C*, *T* and *H* represents a different digit.

$$\overline{SAIMC} + \overline{SAIMC} = \overline{MATHS},$$

where $\overline{SAIMC}$ and $\overline{MATHS}$ are five-digit numbers. Find the sum of all of the different possible values of *T*.