SOUTH AFRICAN INTERNATIONAL MATHEMATICS COMPETITION

Durban • 1 to 6 August 2019

Elementary Mathematics International Contest

Team Contest

Time limit: 70 minutes

Information:

- You are allowed 70 minutes for this paper, consisting of 10 questions printed on separate sheets. For questions 1, 3, 5, 7 and 9, only numerical answers are required. For questions 2, 4, 6, 8 and 10, full solutions are required.
- Each question is worth 40 points. For odd-numbered questions, no partial credits are given. There are no penalties for incorrect answers, but you must not give more than the number of answers being asked for. For questions asking for several answers, full credit will only be given if all correct answers are found. For even-numbered questions, partial credits may be awarded.
- Diagrams shown may not be drawn to scale.

Instructions:

- Write down your team's name in the space provided on every question sheet.
- Enter your answers in the space provided after the individual questions on the question paper.
- During the first 10 minutes, the four team members examine the first 8 questions together, and altogether discuss them. Then they distribute the questions among themselves, with each team member is allotted at least 1 question.
- During the next 35 minutes, the four team members write down the solutions of their allotted problems on the respective question sheets, with no further communication / discussion among themselves.
- During the last 25 minutes, the four team members work together to write down the solutions of the last 2 questions on the respective questions sheets.
- It is forbidden to use instruments such as protractors, calculators and electronic devices.
- At the end of the contest, you must hand in the envelope containing all question sheets and all scratch papers.

English Version

Team: ______________________ ***Score:*** ______________

For Juries Use Only

No.	1	2	3	4	5	6	7	8	9	10	Total	Sign by Jury
Score												
Score												

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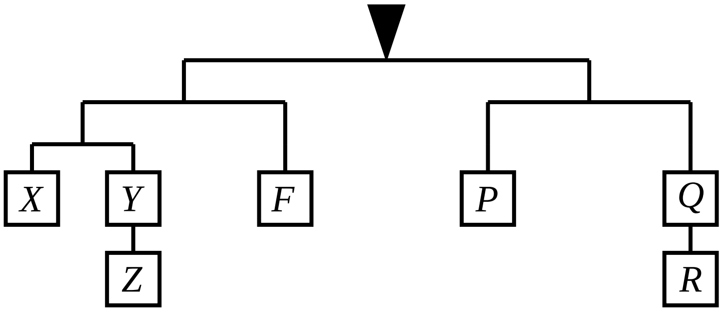
TEAM CONTEST

3rd August, 2019, Durban, South Africa

Team： ______________________ Score： ______________

1. In the figure below, the balance is in equilibrium and each of the weights is an integer greater than zero grams. If the sum of all weights is the largest possible integer not greater than 2019 grams, find the largest possible value, in gram, of *X*.



Answer: ______________________ grams

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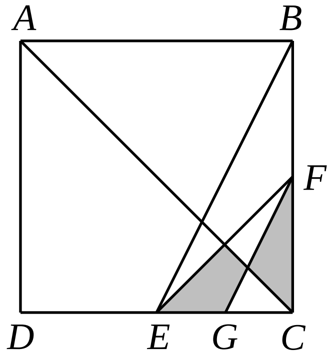
2. In the figure below, *ABCD* is a square and points *E*, *G* and *F* are the midpoints of *CD*, *CE* and *BC* respectively. What is the ratio of the total area of the shaded regions to the area of *ABCD*?



Answer: ____________ : ____________

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3. There are nine cards that are arranged in a sequence 7, 8, 9, 4, 5, 6, 1, 2, 3 in this order as shown below. On each turn, we may remove any number of consecutive cards, reverse their order and put them back anywhere in the row. After three turns the cards are in the sequence 1, 2, 3, 4, 5, 6, 7, 8, 9. What are the three turns? Show your steps.

7	8	9	4	5	6	1	2	3

	7	8	9	4	5	6	1	2	3
→									
→									
→	1	2	3	4	5	6	7	8	9

Answer: ____________________

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4. Peter and Jack went to a store to buy some pens. In the store, they are selling different pen packs such that each pack has exactly 1, 2, 4, 8, 16, … pens. For example, if Peter wants to buy 14 pens, then he needs to buy packs with 8, 4 and 2 pens. It is known that Peter bought *n* pens and Jack bought $n+1$ pens, where $n \le 2019$. If Peter and Jack bought the minimum number of packs of pens and Peter has 4 packs more than Jack, then find the number of different values of *n*.

Answer: ______________________________

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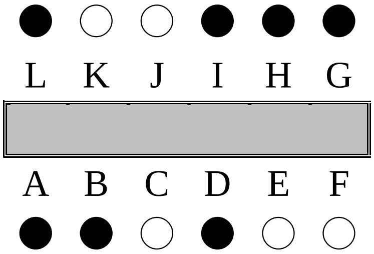
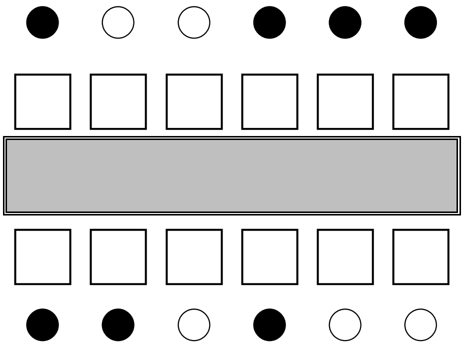
5. Twelve children, A, B, C, D, E, F, G, H, I, J, K and L, sit at a long table with chairs on each side, forming six opposite pairs. Six of them are boys and the other six are girls. The party host stipulates two rules. First, each boy must have a girl on the seat opposite him. Second, if a girl is not in a seat at either end of the table, then there must be a boy on each side of her. However, not every child observed these rules. In the diagram below, those children observing the rules are represented by white circles and those not observing the rules are represented by black circles. Since neither rule applies to a girl sitting on a seat at either end, such a girl may be represented by a white circle or a black circle. What is the gender of each child, represent a girl as “g”, a boy as “b”?





Answer: ______________________

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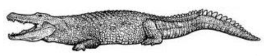
Elementary Mathematics International Contest

TEAM CONTEST

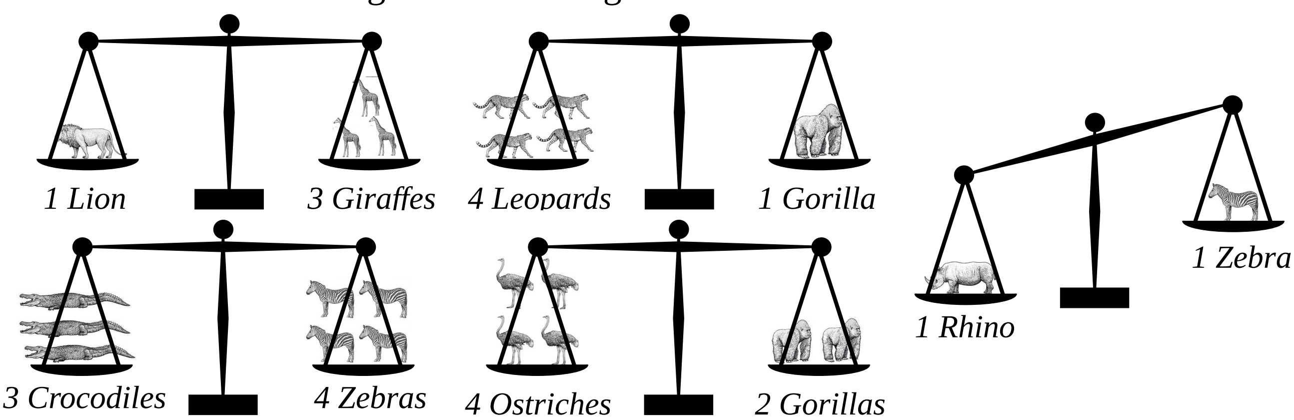
3rd August, 2019, Durban, South Africa

Team：____________________ Score：____________

6. There are 9 kinds of animal figurines in a store:



The weights of these animal figurines are 1 kg, 2kg, 3 kg, 4 kg, 5 kg, 6 kg, 7 kg, 8 kg or 9 kg. Different kinds of animal figurines have different weights and the same kind of animal figurines have the same weight. I put some figurines on a balance scale and get the following result:



What is the weight, in kg, of each kind of animal figurine?

Answer:

1 Crocodile = kg	1 Elephant = kg	1 Giraffe = kg
1 Gorilla = kg	1 Lion = kg	1 Leopard = kg
1 Ostrich = kg	1 Rhino = kg	1 Zebra = kg

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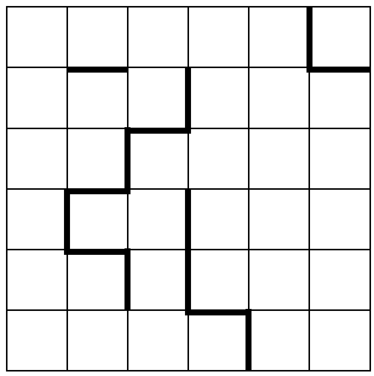
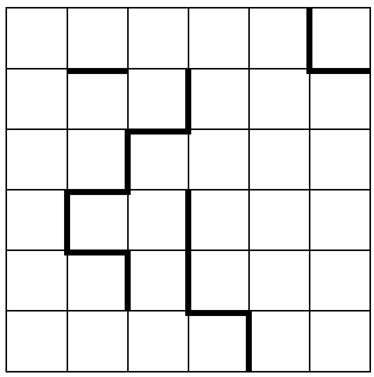
7. In the 6×6 grid below, each row and column contains three kinds of digits: one 1, two 2s and three 3s. A bold line is drawn between two adjacent squares if only if both contain the same number. Now all the numbers have been erased. Complete the grid.





Answer: ____________________

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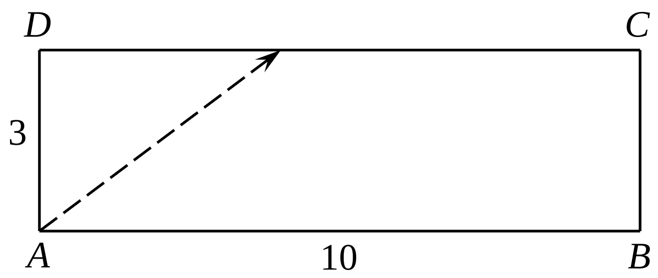
8. A ball is shot from corner *A* of a billiard table *ABCD* so that the first bounce is on the side *CD*. The ball continues to bounce from the sides of the table (by reflection, at a constant speed and without friction). After exactly five bounces (five reflections from the sides of the table), the ball stops in corner *D*. If $AD = 3$ m and $AB = 10$ m, what is the total length, in m, of the longest path covered by the ball as it moves from *A* to *D*.



Answer: ____________________ m

IMC 2019 SOUTH AFRICA

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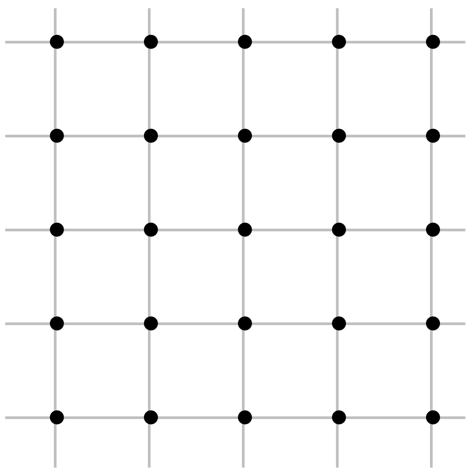
Team： ____________________ Score： ______________

9. The figure shows 25 points, all of which are located in a square grid. How many straight lines can you draw, each containing exactly three of the 25 points?



Answer: ______________________ lines

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10. A special calculator displays the number 1 on its screen when it is switched on and has only two buttons: @ and #. The @ and # buttons work differently:

Button	Effect
@	Multiply by 5, then plus 2
#	Multiply by 7, then minus 5

For example, if you press @ first, then the calculator displays $1\times5+2=7$. Pressing @ again, it displays $7\times5+2=37$. Next, pressing # will then change the display to $37\times7-5=254$.

How many different three-digit numbers can be displayed by pressing @ and # buttons? (Just pressing one button many times is allowed.)

Answer: ____________ three-digit numbers