SOUTH AFRICAN INTERNATIONAL MATHEMATICS COMPETITION

Durban • 1 to 6 August 2019

Invitational World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes

Information:

- You are allowed 120 minutes for this paper, consisting of 12 questions in Section A to which only numerical answers are required, and 3 questions in Section B to which full solutions are required.
- Each question in Section A is worth 5 points. No partial credits are given. There are no penalties for incorrect answers, but you must not give more than the number of answers being asked for. For questions asking for several answers, full credit will only be given if all correct answers are found. Each question in Section B is worth 20 points. Partial credits may be awarded.
- Diagrams shown may not be drawn to scale.

Instructions:

- Write down your name, your contestant number and your team's name in the space provided on the first page of the question paper.
- For Section A, enter your answers in the space provided after the individual questions on the question paper. For Section B, write down your solutions on spaces provided after individual questions.
- You must use either a pencil or a ball-point pen which is either black or blue.
- You may not use instruments such as protractors, calculators and electronic devices.
- At the end of the contest, you must hand in the envelope containing the question paper and all scratch papers.

English Version

Team:_____________ ***Name***:_________________ ***No.***: ____________ ***Score:***_________

For Juries Use Only

No.	Section A												Section B			Total	Sign by Jury
	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3		
Score																	
Score																	

Section A.

In this section, there are 12 questions. Fill in the correct answer in the space provided at the end of each question. Each correct answer is worth 5 points.

1. Function F, when applied to any four-digit positive integer $\overline{abcd}$, produces another positive integer, according to the rule: $F(\overline{abcd}) = a^4 + b^3 + c^2 + d^1$.
   For example, $F(2019) = 2^4 + 0^3 + 1^2 + 9^1 = 26$.
   Find the value of $F(2019) - F(2018) + F(2017) - F(2016) + ... - F(2000)$.

   Answer : ____________

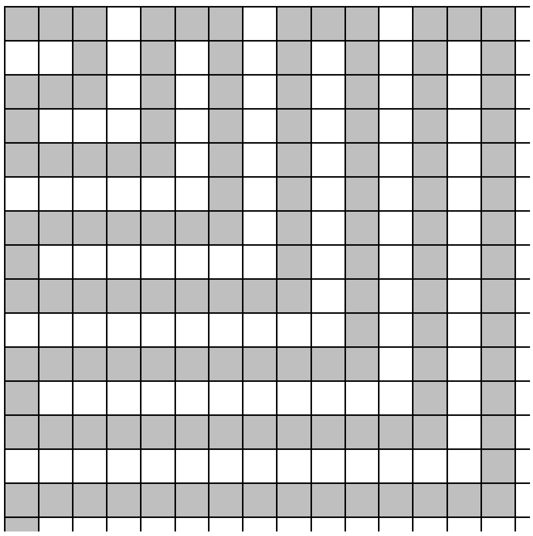
2. What is the smallest positive integer n such that $55n^3$ has exactly 55 positive divisors (including 1 and itself)?

   Answer : ____________

3. Three boxes A, B and C, contain 100, 50 and 80 marbles of the same size respectively, some of which are black. In box A there are 15 black marbles. We select a box at random and then take a marble from that box, again at random. The probability to obtain a black marble this way is $\frac{101}{600}$. What is the greatest possible number of black marbles in box C?

   Answer : ________ black marbles

4. The top-left portion of a grid of 101×101 white squares is shown below. A chain is formed by coloring squares grey as shown. The chain starts at the upper left-hand corner and goes on until it cannot go on any further. In total, how many squares are colored grey in the entire grid of 101×101 squares?



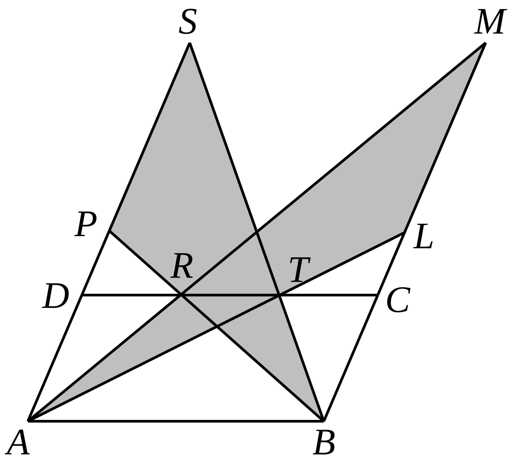
   Answer: ________ squares

5. How many different three-digit positive integers are there that can be expressed as a sum of exactly nine different powers of 2?

   Answer : ____________

6. In the figure below, points R and T lie on the side CD of the parallelogram $ABCD$

such that $DR = RT = TC$. Lines AR and AT intersect the extension of BC at points M and L respectively, and the lines BT and BR intersect the extension of AD at points S and P respectively. If the area of the parallelogram $ABCD$ is 48 cm^2, then what is the area, in cm^2, of the shaded region?



Answer : __________ cm^2

7. There are some positive integer pairs (x, y) that satisfy $\dfrac{1}{\sqrt{x}} + \dfrac{1}{\sqrt{y}} = \dfrac{1}{\sqrt{20}}$.

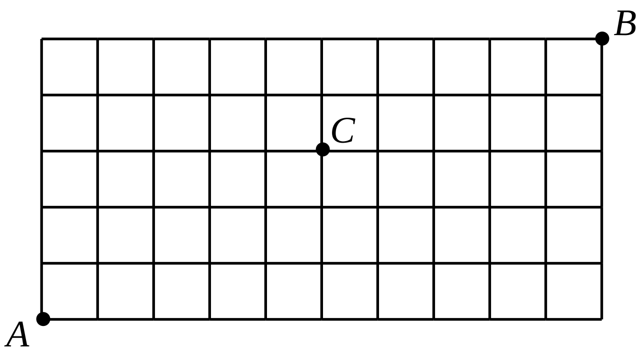
   How many different possible values of the product of x and y are there?

Answer : ______________

8. There are some integer pairs (m, n) that satisfy $\dfrac{(m^2 + mn + n^2)}{(m + 2n)} = \dfrac{13}{3}$. Find the value of $m + 2n$.

Answer : ______________

9. In the figure below, an ant starting at point A is travelling along the grid lines of the 10×5 grid. It is only allowed to go up and to the right and is not allowed to go through point C. How many ways are there for the ant to go from point A to point B?



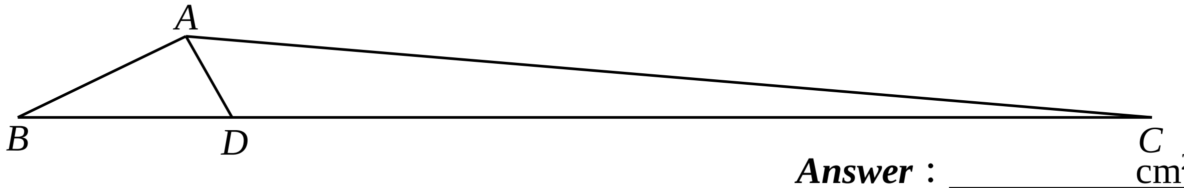
Answer : ________ ways

10. Given that f is a function from the non-negative real numbers to the non-negative

real numbers such that $f(a^3)+f(b^3)+f(c^3)=3f(a)f(b)f(c)$ and $f(1)\neq 1$, where a, b and c are non-negative real numbers. Find $f(2019)$.

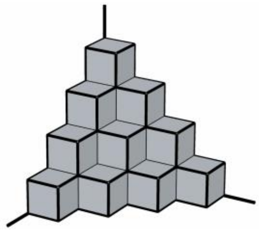
Answer : ____________

11. In the figure below, ABC is a triangle such that $\angle BAC=150°$, $BC=74$ cm and point D is on BC such that $BD=14$ cm. If $\angle ADB=60°$, then what is the area, in cm², of triangle ABC?



Answer : ____________ cm²

12. In the figure below, there is a certain number of cubes that are piled on top of each other to form a triangular tower that fits one corner of a room. If we use exactly 1330 identical cubes, then how many levels does the tower have? Note that not all cubes can be seen from this view.



Answer : ____________ levels

Section B.

Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. The increasing sequence 1, 3, 4, 9, 10, 12, 13, … consists of all the positive integers which can be expressed as powers of 3 or sums of distinct powers of 3. Find the 100th term of this sequence.

Answer : ____________

2. Find all the integer pairs (x, y) that satisfy the equation

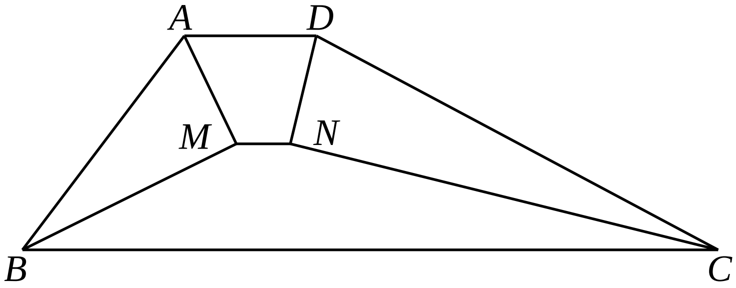
$$7x^2-40xy+7y^2=(|x-y|+2)^3.$$

Answer : ______________________________

Answer : ______________________________

3. In a quadrilateral $ABCD$, $BC // AD$, $BC = 26$ cm, $AD = 5$ cm, $AB = 10$ cm and $CD = 17$ cm. The bisectors of $\angle A$ and $\angle B$ intersect at M while the bisectors of

$\angle C$ and $\angle D$ intersect at N. Find the length, in cm, of MN.



Answer : ____________ cm