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SOUTH AFRICAN INTERNATIONAL MATHEMATICS COMPETITION

Durban • 1 to 6 August 2019

Invitational World Youth Mathematics Intercity Competition

Team Contest

Time limit: 70 minutes

Information:

- You are allowed 70 minutes for this paper, consisting of 10 questions printed on separate sheets. For questions 1, 3, 5, 7 and 9, only numerical answers are required. For questions 2, 4, 6, 8 and 10, full solutions are required.
- Each question is worth 40 points. For odd-numbered questions, no partial credits are given. There are no penalties for incorrect answers, but you must not give more than the number of answers being asked for. For questions asking for several answers, full credit will only be given if all correct answers are found. For even-numbered questions, partial credits may be awarded.
- Diagrams shown may not be drawn to scale.

Instructions:

- Write down your team's name in the space provided on every question sheet.
- Enter your answers in the space provided after the individual questions on the question paper.
- During the first 10 minutes, the four team members examine the first 8 questions together, and altogether discuss them. Then they distribute the questions among themselves, with each team member is allotted at least 1 question.
- During the next 35 minutes, the four team members write down the solutions of their allotted problems on the respective question sheets, with no further communication / discussion among themselves.
- During the last 25 minutes, the four team members work together to write down the solutions of the last 2 questions on the respective questions sheets.
- It is forbidden to use instruments such as protractors, calculators and electronic devices.
- At the end of the contest, you must hand in the envelope containing all question sheets and all scratch papers.

English Version

Team: _____ ***Score:*** _____

For Juries Use Only

No.	1	2	3	4	5	6	7	8	9	10	Total	Sign by Jury
Score												
Score												

SOUTH AFRICAN INTERNATIONAL MATHEMATICS COMPETITION

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Invitational World Youth Mathematics Intercity Competition

TEAM CONTEST

3rd August, 2019, Durban, South Africa

Team：_____ Score：_____

1. Find all triples of positive integers (I, M, C), where I, M and C are prime numbers and $I \le M \le C$, such that $I \times M \times C = I + M + C + 1007$.

Answer: _____

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TEAM CONTEST

3rd August, 2019, Durban, South Africa

Team：_____ Score：_____

2. *N* integers from 2021 consecutive positive integers are chosen such that the difference between any two of them is not a prime number. Find the largest value of *N*?

Answer: _____

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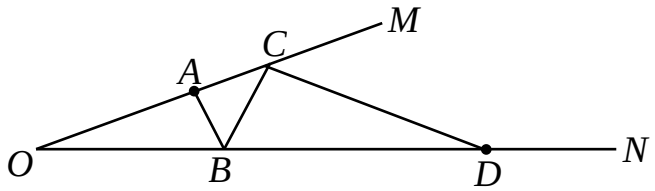
Team：_____ Score：_____

3. In the figure below, points A, C are on ray OM and B, D are on ray ON. It is given that $OA = 6$ cm, $OD = 16$ cm and $\angle NOM = 20°$. What is the minimum length, in cm, of $AB + BC + CD$?



Answer: _____ cm

Invitational World Youth Mathematics Intercity Competition

TEAM CONTEST

3rd August, 2019, Durban, South Africa

Team : _____ Score : _____

4. Determine the number of ordered pairs of digits (a, b) such that when the number $\overline{2a1b9}^{2019}$ is divided by 13, it leaves a remainder of 1. ($\overline{2a1b9}$ is a five-digit number.)

Answer: _____ pairs

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Team : _____ Score : _____

5. There are 10 identical red balls, 15 identical black balls and 20 identical white balls. We want to distribute all the balls to 2 boys and a girl. Each boy must receive at least 2 balls of each color, and the girl must receive at least 3 balls of each color. What is the total number of different ways to distribute the balls to those three children?

Answer: _____ ways

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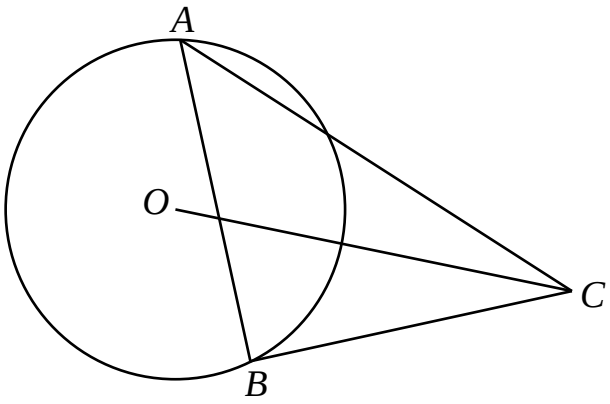
Team : _____ Score : _____

6. Point B is an arbitrary point on a circle with centre O and radius 1 cm. ABC is a triangle with A on the circle such that $AB = BC$ and $\angle ABC = 90°$. Determine the maximum length, in cm, of OC.



Answer: _____ cm

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Team : _____ Score : _____

7. Each of 14 girls has a different juicy piece of good news, and is eager to share it with all the other girls. In each round, some or all of the girls engage in phone conversations in pairs, telling each other all the pieces they have heard.

(a). What is the minimum number of rounds required in order for every girl to have heard all the pieces of good news? (10 points)

(b). Show all calls in each round to achieve the minimum number of rounds? (30 points)

Answer: (a). _____ rounds

(b).

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8. On the plane, 5 points X_1, X_2, X_3, X_4 and X_5 are chosen, and each pair of these points is connected by a blue line. Suppose no two blue lines are parallel or perpendicular to each other. Now for each point X_i $(1 \le i \le 5)$ and each blue line L not passing through X_i, a red line is drawn passing through X_i and perpendicular to L. What is the maximum number of points of intersection formed by the red lines?

Answer: _____ points

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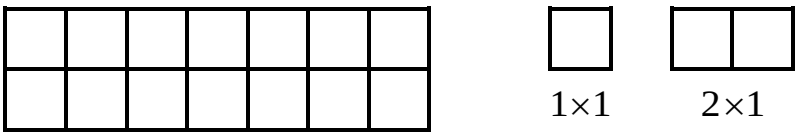
9. A wall with dimensions 2×7 squares has to be covered with ceramic tiles. There are two types of ceramic tiles that are available: the 1×1 (identical) tile and the 2×1 (identical) tile. The 2×1 tile can be rotated before being placed on the board. We are provided with as many tiles of each type as we need. In how many ways can we cover the 2×7 wall with such tiles?



Answer: _____ ways

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Team：_____ Score：_____

10. For any positive integer x, let $S(x)$ be the sum of all the digits of x in its decimal representation. Find all solutions of the equation $x=(S(x)+9)^2$.

Answer: _____