Indonesia International Mathematics Competition 2021 (Virtual)

Indonesia, 27^{th} July to 1^{st} August 2021

Invitational World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes

Information:

- You are allowed 120 minutes for this paper, consisting of 12 questions in Section A to which only numerical answers are required, and 3 questions in Section B to which full solutions are required.
- Each question in Section A is worth 5 points. No partial credits are given. There are no penalties for incorrect answers, but you must not give more than the number of answers being asked for. For questions asking for several answers, full credit will only be given if all correct answers are found. Each question in Section B is worth 20 points. Partial credits may be awarded.
- Diagrams shown may not be drawn to scale.

Instructions:

- Write down your name, your contestant number and your team's name on the answer sheet.
- For Section A, enter your answers in the space provided on the answer sheet.
  For Section B, write down your full solutions on spaces provided on the answer sheet.
- You must use either a HB, B or 2B pencil or a ball-point pen which is either black or blue.
- You may not use instruments such as protractors, calculators and electronic devices.
- At the end of the contest, you must hand in the envelope containing the question paper, the answer sheet and all scratch papers.

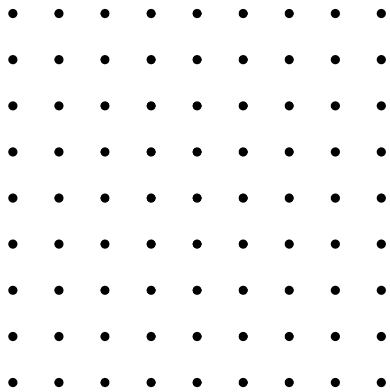
English Version

Team: ______________ ***Name:*** __________________ ***ID.:*** _____________ __________

Section A.

In this section, there are 12 questions. Fill in the correct answer in the space provided at the end of each question. Each correct answer is worth 5 points.

1. There are 81 dots which are arranged in a 9×9 array as shown below. If the distance between any two adjacent dots on the same row or column is 1 cm, determine the number of rectangles that can be formed having an area of 12 cm^2, where all four vertices are among these 81 dots.



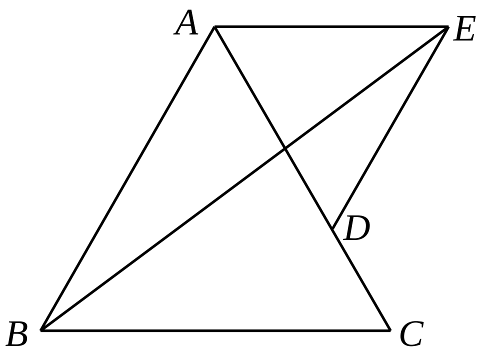
2. There are 99 numbers that are placed around a circle, each one is either 1 or -1. The product of each block of 10 adjacent numbers along the circle is then computed. Let S denote the sum of these 99 products. If there is at least one instance of 1 and at least one instance of -1 around the circle, what is the difference between the maximum value and the minimum value of S?

3. Let $P(x)$ and $Q(x)$ be two quadratic polynomials with integer coefficients and their leading coefficients are both 1. It is known that $P(Q(0)) = Q(P(0)) = 1$ and $P(0) + Q(0) = 2$. Find the value of $P(3) + Q(3)$.

4. Let a_1, a_2, a_3, …, a_9 be a random arrangement of the numbers 1, 2, 3, …, 9. What is the greatest possible value of

$$|a_1 - \sqrt{3}a_2| + |a_2 - \sqrt{3}a_3| + |a_3 - \sqrt{3}a_4| + ... + |a_8 - \sqrt{3}a_9| + |a_9 - \sqrt{3}a_1|?$$

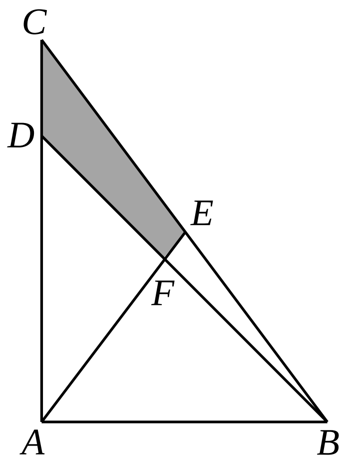
5. In the diagram below, *ABC* and *ADE* are both equilateral triangles having side length of 6 cm and 4 cm, respectively. Find the length, in cm, of *BE*.



6. Let *x* and *y* be real numbers such that $(2x + \sqrt{1 + 4x^2})(3y + \sqrt{1 + 9y^2}) = 1$.

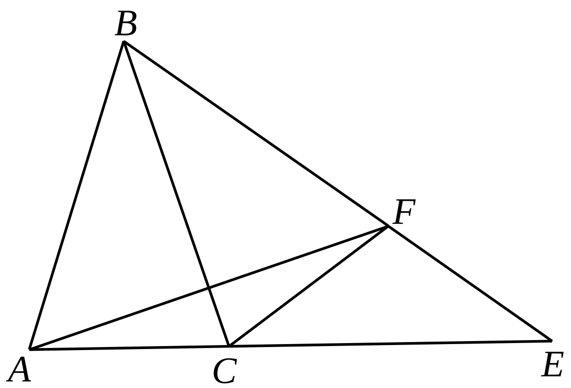
Find the numerical value of $(2x + 3y)^2$.

7. In the diagram below, A is a right angle, $AB = 3$ cm, $BC = 5$ cm and $CD = 1$ cm. If $BE = EC$, then what is the area, in cm^2, of the shaded region?



8. If a prime number can be written in the form $k^k + 1$, where k is a positive integer, then such a prime number is called an *IMC prime number*. Find the largest *IMC prime number* not exceeding 20212021.

9. In an isosceles triangle ABC, where $AB = BC$, point E is on the extension of side AC, where C is between A and E and point F is on the segment BE such that $AC = CF = FE$ and $\angle BAF = 3\angle FAE$, as shown in the diagram below. Find the measure, in degrees, of $\angle FAE$.



10. Find the greatest positive integer n such that $n+3$ divides $1^3 + 2^3 + ... + n^3$.

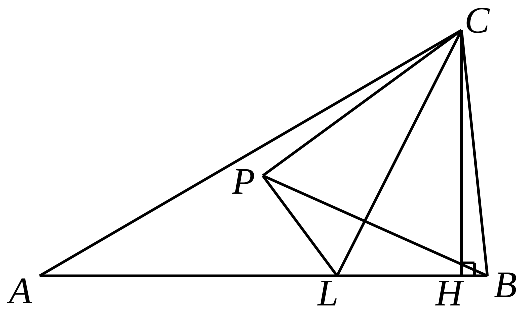
11. The nation of *IMC-land* consists of 8 islands, none of which are connected to each other. Since each citizen wants to visit each of the other islands, the government plans to build bridges between the islands. However, each island has a volcano that could erupt at any time, destroying that island and any bridges that are connected to it. The government wants to guarantee that after any eruption, a citizen from any of the remaining 7 islands can go on a tour, visiting each of the remaining islands exactly once and returning to their home island (only at the end of the tour). What is the minimum number of bridges needed to be built?

12. Given that $\dfrac{(a-b)(b-c)(c-a)}{(a+b)(b+c)(c+a)} = \dfrac{1}{2021}$, find the value of $\dfrac{a}{a+b} + \dfrac{b}{b+c} + \dfrac{c}{c+a}$.

Section B.

Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. Let P be an interior point of acute triangle ABC such that $CP = BP$ and $\angle BPC = 2\angle BAC$. Let the angle bisector of $\angle ACB$ and AB intersect at L and point H be on AB such that $CH \perp AB$, where L is between points A and H, as shown in the diagram below. If $CP = CH = 28$ cm and the area of triangle CPL is 196 cm^2, then find the length, in cm, of LH.



2. In a class of 14 boys and 17 girls, some candies are to be distributed. Every boy receives an equal number of candies as each other, while every girl also receives an equal number of candies as each other. Each person receives at least one candy, but the number of candies received by each boy may be different from that received by each girl. If the total number of candies cannot be redistributed in any other way to satisfy our conditions, determine the maximum number of candies that we can have.

3. Let M and N be non-negative integers such that C_{1010}^{2021} is divisible by $2^M \times 3^N$. Find the sum of all the possible values of $M + N$. (Note: C_{1010}^{2021} refers to the combination of 2021 things taken 1010 at a time without repetition.)