Indonesia International Mathematics Competition 2021 (Virtual)

Indonesia, 27th July to 1st August 2021

Invitational World Youth Mathematics Intercity Competition

Team Contest

Time limit: 70 minutes

Information:

- You are allowed 70 minutes for this paper, consisting of 10 questions printed on separate sheets. For questions 1, 3, 5, 7 and 9, only numerical answers are required. For questions 2, 4, 6, 8 and 10, full solutions are required.
- Each question is worth 40 points. For odd-numbered questions, no partial credits are given. There are no penalties for incorrect answers, but you must not give more than the number of answers being asked for. For questions asking for several answers, full credit will only be given if all correct answers are found. For even-numbered questions, partial credits may be awarded.
- Diagrams shown may not be drawn to scale.

Instructions:

- Write down your team's name in the space provided on every question sheet.
- Enter your answers in the space provided after the individual questions on the question paper.
- During the first 10 minutes, the four team members examine the first 8 questions together, and altogether discuss them. Then they distribute the questions among themselves, with each team member is allotted at least 1 question.
- During the next 35 minutes, the four team members write down the solutions of their allotted problems on the respective question sheets, with no further communication / discussion among themselves.
- During the last 25 minutes, the four team members work together to write down the solutions of the last 2 questions on the respective questions sheets.
- It is forbidden to use instruments such as protractors, calculators and electronic devices.
- At the end of the contest, you must hand in the envelope containing all question sheets and all scratch papers.

English Version

Team: _____ _____

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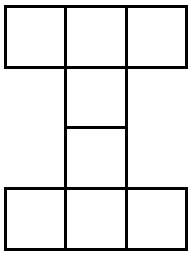
TEAM CONTEST

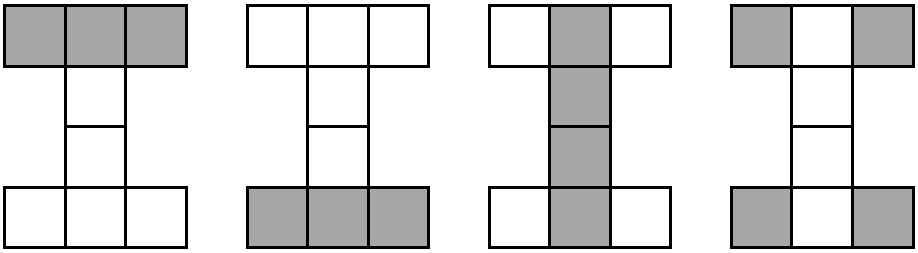
29th July, 2021, Indonesia

Team : _____ Solver : _____ ID : _____

1. Consider the 'I' shaped diagram below:



Each square is filled with exactly one number from the following eight numbers -1, -1, -1, 0, 0, 1, 1 and 1 such that the sum of the numbers in each of the rows, column and four corners are all the same. How many ways are there to fill the diagram?



Answer: _____ ways

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2. Let x and y be real numbers such that:

$$\begin{cases} x^2 + 5xy + y^2 = 7 \\ x^2y + xy^2 = 2 \\ x + y \neq 2 \end{cases}$$

Find all possible values of $x^2 + y^2$.

Answer: _____

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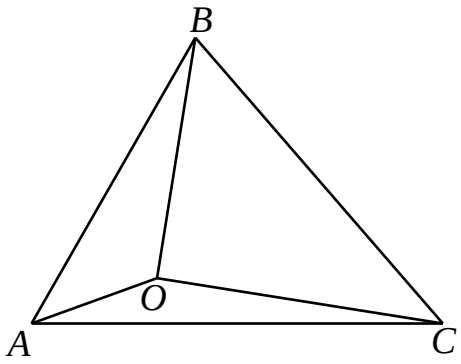
3. Let ABC be a triangle and O be a point in its interior such that $\angle ABO = \angle CAO$, $\angle BAO = \angle BCO$ and $\angle BOC = 90°$, as shown in the diagram below. If $AC = 2$ cm, then find the length, in cm, of OC.



Answer: _____ cm

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4. Find all four-digit integers m that are less than 2021 for which there exists a positive integer n such that $m-n$ is a positive prime number and $m \times n$ is a perfect square.

Answer: _____

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5. Points A_1, A_2, …, A_{100} are on a line, in this order, such that $A_1A_2 = \frac{1}{1\times 2}$ cm, $A_2A_3 = \frac{1}{2\times 3}$ cm, $A_3A_4 = \frac{1}{3\times 4}$ cm, …, $A_{99}A_{100} = \frac{1}{99\times 100}$ cm. If the segment A_mA_n, where $1 \le m < n \le 100$, has a length of $\frac{1}{15}$ cm, then what is the largest possible value of $m+n$ that satisfies the given conditions?

Answer: _____

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6. Using four different colours, in how many ways can we paint the integers 1, 2, 3, ..., 10 such that every pair of integers whose difference is a prime number must have different colours?

1 2 3 4 5 6 7 8 9 10

Answer: _____ ways

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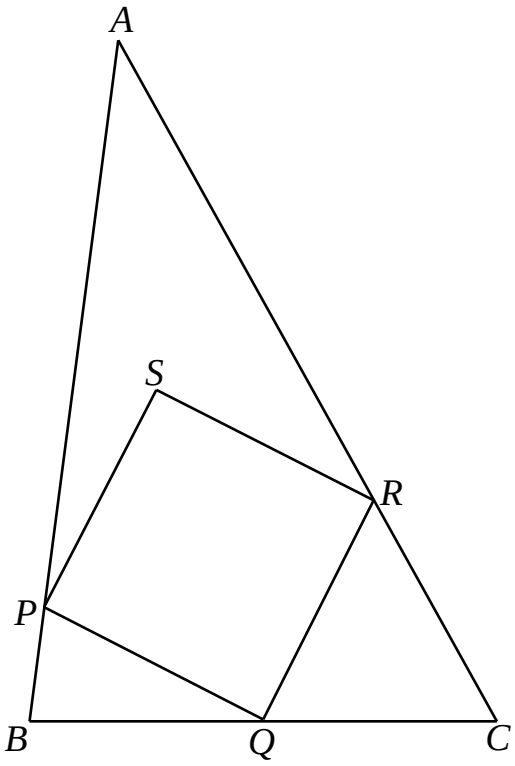
7. In the diagram below, ABC is a triangle where $AB = 60$ cm and $AC = 68$ cm. $PQRS$ is a square where $AP = 50$ cm, $AR = 46$ cm and Q is the mid-point of BC. What is the ratio of the area of $PQRS$ to the area of ABC?



Answer: _____ :

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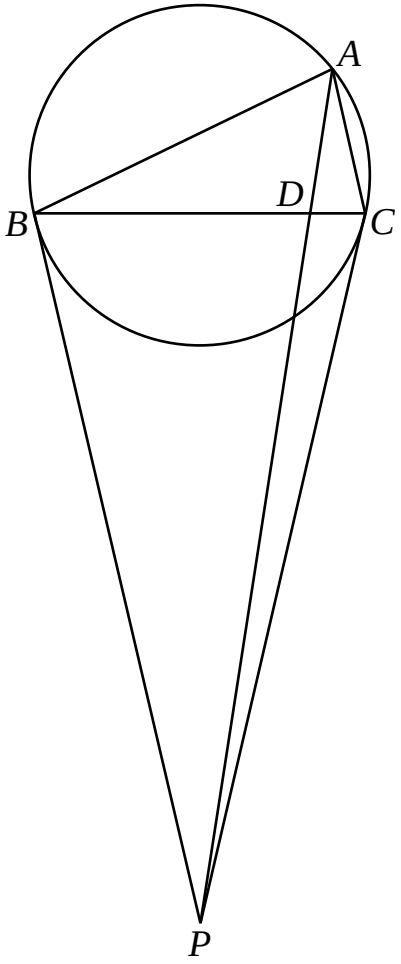
8. Let triangle ABC be an acute triangle inscribed inside circle O. Let P be a point outside O such that PB and PC are both tangent to O. If AP and BC intersect at D and $\frac{BD}{CD} = 5$, then what is the value of $\frac{AB}{AC}$?



Answer: _____

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9. A positive integer $n>1$ is called an *interesting number* if it satisfies the property:

If p is a prime divisor of n, then $2p+1$ is a divisor of n.

Find the total number of positive divisors of the smallest *interesting number*.

Answer: _____

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10. In how many different ways can we express 2021 as a sum using the numbers 1, 2, 4, 8, 16, 32, 64, 128, 256, 512 and 1024, where it is only allowed to have up to two occurrences of any of these eleven numbers? The order of summation does not matter. For example, $2+1+1$ and $1+2+1$ are considered the same way to express 4.

Answer: _____ ways