Indonesia International Mathematics Competition 2022 (Virtual)

Indonesia, 30th June to 6th July 2022

Invitational World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes

Information:

- You are allowed 120 minutes for this paper, consisting of 12 questions in Section A to which only numerical answers are required, and 3 questions in Section B to which full solutions are required.
- Each question in Section A is worth 5 points. No partial credits are given. There are no penalties for incorrect answers, but you must not give more than the number of answers being asked for. For questions asking for several answers, full credit will only be given if all correct answers are found. Each question in Section B is worth 20 points. Partial credits may be awarded.
- Diagrams shown may not be drawn to scale.

Instructions:

- Write down your name, your contestant number and your team's name on the answer sheet.
- For Section A, enter your answers in the space provided on the answer sheet.
  For Section B, write down your full solutions on spaces provided on the answer sheet.
- You must use either a HB, B or 2B pencil or a ball-point pen which is either black or blue.
- You may not use instruments such as protractors, calculators and electronic devices.
- At the end of the contest, you must hand in the envelope containing the question paper, the answer sheet and all scratch papers.

English Version

Team: _____ ***Name:*** _____ ***ID.:*** _____

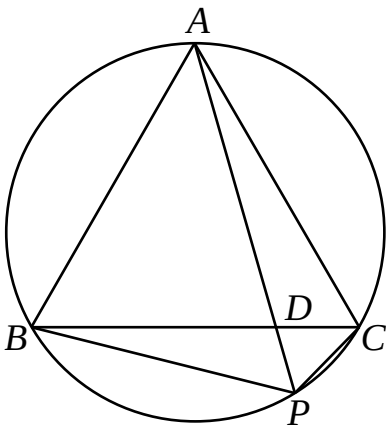
Section A.
In this section, there are 12 questions. Fill in the correct answer in the space provided at the end of each question. Each correct answer is worth 5 points.

1. It is known that the equation $x^2 + px + q = 0$ has two positive integer roots. If $p + q = 16$, then what is the value of q?

2. How many positive integers n are there such that $n^2 + n$ has exactly 6 positive divisors?

3. Let a, b and c be positive real numbers such that the expression $\dfrac{3a^2 + b^2 + 3c^2}{ab + bc + ac}$ attains its minimum value. If $abc = 432$, then find the value of $3a + b + 3c$.

4. In the diagram below, let P be a point on the circumcircle of the equilateral triangle ABC such that $PB = 24\,\text{cm}$ and $PC = 8\,\text{cm}$. If AP and BC intersect at D, then find the length, in cm, of PD.

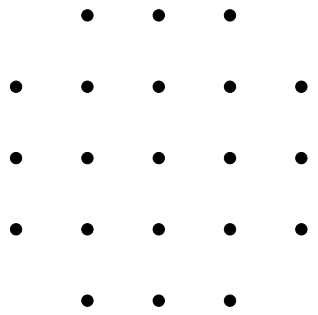


5. Let a, b, c and d be positive integers such that $0 < a < b < c < d < 2022$, $a + d = b + c$ and $bc - ad = 2021$. How many ordered quadruples (a, b, c, d) are there?

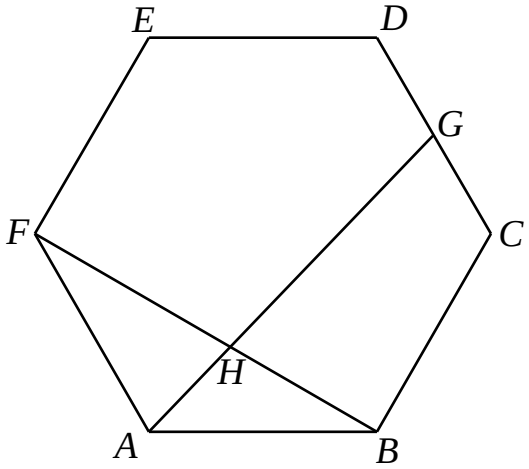
6. Let $f(n)$ denote the number of occurrences of the digit 2 in the following positive integer sequence 1, 2, 3, 4, …, n. For example: $f(23) = 7$, as the digit 2 occurs once in each of the numbers 2, 12, 20, 21, 23 and twice in the number 22, so $f(23) = 5 + 2 = 7$. Find a positive integer n such that $f(n) = n$.

7. Let a and b be the two distinct roots of $x^2 + 2018x + 1 = 0$, while c and d be the two distinct roots of $x^2 - 2022x + 1 = 0$.
Find the value of $(a + c)(a - d)(b + c)(b - d)$.

8. From the grid shown in the diagram below, in how many ways can we choose three different points, such that the three chosen points form a triangle?



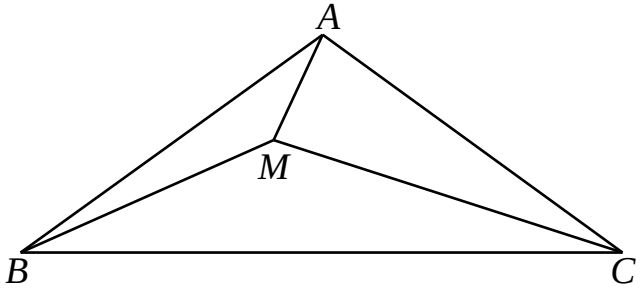
9. In the diagram below, let *ABCDEF* be a regular hexagon. Let *G* be the midpoint of *CD* and *H* be the intersection point of *AG* and *BF*, as shown in the diagram below. If the length of *BF* is 140 cm, then find the length, in cm, of *BH*.



10. One thousand non-zero numbers $x_1,\ x_2,\ x_3,\ x_4, \ldots,\ x_{1000}$ are written in a cyclic order around a circle. It turns out that each number that is written in an odd position is equal to the sum of its two neighbors and that each number that is written in an even position is equal to the product of its two neighbors. What are the possible values for the sum of these one thousand numbers?

11. How many 10-digit positive integers are there such that the product of their digits is 120 and the sum of their digits is 20?

12. In the diagram below, triangle *ABC* is isosceles with $AB = AC$ and $\angle A = 108°$. Let *M* be a point in the interior of *ABC* such that $\angle MAB = 30°$ and $\angle MBA = 12°$. Find the angle measure, in degrees, of $\angle MCB$.



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Section B.

Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. Anna and Boris play a game using a 6×8 board that contains one token on each unit square. Anna goes first, and they take alternating turns thereafter. In her turn, Anna takes two tokens on adjacent squares in the same row or column. In his turn, Boris takes only one token. However, if Anna can no longer take two adjacent tokens when her turn comes, Boris will get all the remaining tokens. What is the maximum number of tokens Boris is guaranteed to get?

Answer: _____

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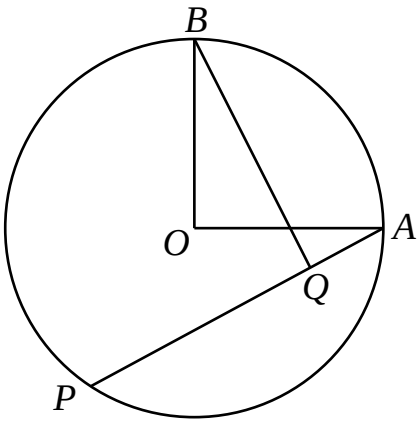
2. Let OA and OB be two radii of a circle with centre O, such that $OA \perp OB$, as shown in the diagram below. Let P be a point on the circumference and Q be the point on AP such that $AP = 4AQ$. If $OA = 8$ cm, then what is the minimum possible length, in cm, of BQ?



Answer: _____ cm

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3. Let $\overline{abcd}$ be a four-digit number, where $a, c \neq 0$, such that $\dfrac{\sqrt{\overline{abcd}}}{\sqrt{\overline{ab}}+\sqrt{\overline{cd}}}$ is a rational number. Find all possible value(s) of $\overline{abcd}$.

Answer: _____