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petition Indonesia, 30<sup>th</sup> June to 6<sup>th</sup> July 2022

Invitational World Youth Mathematics Intercity Competition

# **Team Contest**

### Time limit: 70 minutes

#### Information:

- You are allowed 70 minutes for this paper, consisting of 10 questions printed on separate sheets. For questions 1, 3, 5, 7 and 9, only numerical answers are required. For questions 2, 4, 6, 8 and 10, full solutions are required.
- Each question is worth 40 points. For odd-numbered questions, no partial credits are given. There are no penalties for incorrect answers, but you must not give more than the number of answers being asked for. For questions asking for several answers, full credit will only be given if all correct answers are found. For even-numbered questions, partial credits may be awarded.
- Diagrams shown may not be drawn to scale.

#### **Instructions:**

- Write down your team's name in the space provided on every question sheet.
- Enter your answers in the space provided after the individual questions on the question paper.
- During the first 10 minutes, the four team members examine the first 8 questions together, and altogether discuss them. Then they distribute the questions among themselves, with each team member is allotted at least 1 question.
- During the next 35 minutes, the four team members write down the solutions of their allotted problems on the respective question sheets, with no further communication / discussion among themselves.
- During the last 25 minutes, the four team members work together to write down the solutions of the last 2 questions on the respective questions sheets.
- It is forbidden to use instruments such as protractors, calculators and electronic devices.
- At the end of the contest, you must hand in the envelope containing all question sheets and all scratch papers.

# **English Version**

Team:



**1.** A palindromic number is a number that is the same when we read it forward and backward. How many 11-digit palindromic numbers are divisible by 101?



### Indonesia International Mathematics Competition 2022 (Virtual) Indonesia, 30<sup>th</sup> June to 6<sup>th</sup> July 2022 International Mathematics Competition Invitational World Youth Mathematics Intercity Competition **Team Contest**

2<sup>nd</sup> July, 2022, Indonesia Solver : ID: Team :

The function f(x) is defined for every real number x and satisfies the following 2. condition:  $f(x) + 2f(y) = 3f(\frac{x+2y}{3})$  for every two real numbers x and y. If f(2)=1 and f(5)=7, then find f(2022).



# Invitational World Youth Mathematics Intercity Competition

### **Team Contest** 2<sup>nd</sup> July, 2022, Indonesia

Solver :

Team :

ID:

**3.** An Indonesian high-tech company has invented an airbag that can be dropped from tall buildings without bursting. An engineer will be conducting the test at the Autograph Tower Thamrin Nine, currently the tallest building in Indonesia, which has 75 floors. His goal is to find the highest floor of this building from which the airbag won't burst when dropped. The engineer only made two identical prototypes of this airbag for testing, which can be dropped from various floors of the building. If an airbag doesn't burst after being dropped, it may be reused without suffering any loss of quality. But if both airbags burst before the engineer has determined the highest floor, then the test is considered to be a failure. What is the least number of times the engineer must drop the airbags in order to determine the highest floor?



Invitational World Youth Mathematics Intercity Competition

### **Team Contest** 2<sup>nd</sup> July, 2022, Indonesia

Solver :

Team:

- ID:
- 4. In the diagram below, points  $A_1$ ,  $A_2$ ,  $A_3$ ,  $A_4$ ,  $A_5$  lie on the circumference of a unit circle and  $\angle A_5 A_2 A_4 = \angle A_1 A_3 A_5 = \angle A_2 A_4 A_1 = \angle A_3 A_5 A_2 = 30^\circ$ . Let  $A_2 A_4$  and  $A_3 A_5$ ,  $A_3 A_5$  and  $A_4 A_1$ ,  $A_4 A_1$  and  $A_5 A_2$ ,  $A_5 A_2$  and  $A_1 A_3$  intersect at points  $B_1$ ,  $B_2$ ,  $B_3$ ,  $B_4$ ,  $B_5$ , respectively. Find the area, in square units, of the pentagon  $B_1 B_2 B_3 B_4 B_5$ .



### Answer:



# Invitational World Youth Mathematics Intercity Competition

### **Team Contest** 2<sup>nd</sup> July, 2022, Indonesia

Team :

Solver :

ID:

5. If ax + by = 7,  $ax^2 + by^2 = 49$ ,  $ax^3 + by^3 = 133$  and  $ax^4 + by^4 = 406$ , then find the numerical value of 2022(x + y) + 7(a + b) + 2xy.



# **Team Contest** 2<sup>nd</sup> July, 2022, Indonesia

Team:

Solver :

ID:

**6.** How many different arrangements of 1, 2, 3, 4, 5, 6 and 7 in a row are there such that the sum of any 2 consecutive numbers is a prime?

Answer:

arrangements



# Team Contest $2^{nd}$ July, 2022, IndonesiaTeam :Solver :ID :

7. Let *PQRS* be a trapezoid such that *PS//QR* and *T* is the intersection point of *PR* and *QS*, as shown in the diagram below. Let *I*, *M* and *C* be the midpoints of *ST*, *PQ* and *RT*, respectively. If  $\angle STR = 120^\circ$ ,  $\angle SQR = \angle SPR$  and the area of triangle *IMC* is equal to  $1024\sqrt{3}$  cm<sup>2</sup>, then find the length, in cm, of *PQ*.





# Invitational World Youth Mathematics Intercity Competition

# **Team Contest** 2<sup>nd</sup> July, 2022, Indonesia

Team:

Solver :

ID:

8. In the diagram below, *ABC* is an equilateral triangle, where *M* and *N* are points on *AC* and *BM*, respectively, such that  $\angle ABM = 15^{\circ}$  and  $\angle BAN = 30^{\circ}$ . Let *P* be the intersection point of the line *AN* with the circumcircle of triangle *ABM*. Prove that AN = NP.





# Invitational World Youth Mathematics Intercity Competition

# **Team Contest**

2<sup>nd</sup> July, 2022, Indonesia

Team :

Solver :

ID:

- **9.** Determine how many ordered pairs of integer numbers (*a*, *b*) satisfy the following conditions:
  - $1 \le a, b \le 100$  and  $a \ne b$ .
  - a is even.
  - (a-1)|(b-1), a|b and (a+1)|(b+1)

Answer:

ordered pairs



# Indonesia International Mathematics Competition 2022 (Virtual)

Indonesia,  $30^{\text{th}}$  June to  $6^{\text{th}}$  July 2022

# Invitational World Youth Mathematics Intercity Competition

# **Team Contest**

2<sup>nd</sup> July, 2022, Indonesia

Team:

ID:

**10.** Andy constructed the smallest subset *S* of positive integers that satisfies the following properties:

Solver :

- If number t is in S, then the numbers 6t and 6t + 1 are also in S.
- The number 1 is in *S*, but the numbers 2, 3, 4 and 5 are not in *S*.

Then, Andy calculates the sum of two distinct numbers in *S*, where the two numbers are both less than 2022. How many different possible sums can Andy obtain?