



*Bulgaria International
Mathematics Competition 2023
(Virtual)
Bulgaria, 1st to 7th July 2023*

Elementary Mathematics International Contest

Team Contest

Time limit: 70 minutes

Information:

- You are allowed 70 minutes for this paper, consisting of 10 questions printed on separate sheets. For questions 1, 3, 5, 7 and 9, only numerical answers are required. For questions 2, 4, 6, 8 and 10, full solutions are required.
- Each question is worth 40 points. For odd-numbered questions, no partial credit is given. There are no penalties for incorrect answers, but you must not give more than the number of answers being asked for. For questions asking for several answers, full credit will only be given if all correct answers are found. For even-numbered questions, partial credits may be awarded.
- Diagrams shown may not be drawn to scale.

Instructions:

- Key in your name, your contestant ID and your team's name before going to next page. Write down your team's name in the space of every question sheet.
- You may not use instruments such as protractors, calculators and electronic devices.
- For the odd-numbered questions, Enter your answers in the column provided after each question. For all answers, **no need to key in their units**. The format as following:
 1. For decimal $a.bc$, where a, b and c are digits, key in $a.bc$.
 2. For fraction $\frac{a}{b}$, where a and b are coprime, key in a/b (For example, if your answer is $3\frac{2}{5}$, please key in **17/5**).
 3. For ratio $a : b$, key in $a:b$ or $a;b$ (no need space after “:” or “;”).
 4. For number pair (a, b, c, \dots) , key in a,b,c, \dots (no need space after “,”).
- For even-numbered questions, write your full solution on the blank papers provided.
- During the first 10 minutes, the four team members examine the first 8 questions together, and altogether discuss them. Then they distribute the questions among themselves, with each team member is allotted at least 1 question.
- During the next 35 minutes, the four team members write down the solutions of their allotted problems, with no further communication / discussion among themselves.
- During the last 25 minutes, the four team members work together to solve the last 2 questions.
- At the end of each part contest, you must click “send” for the odd-numbered questions and scan or take a photo of your even-numbered questions solutions then upload to the given website.

English Version

Team: _____



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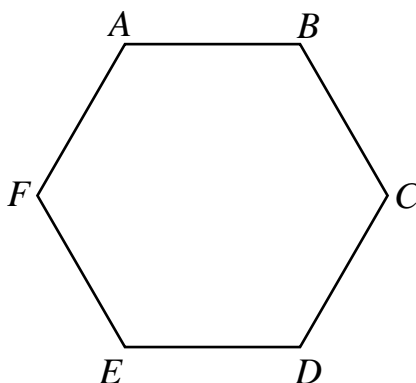
3rd July, 2023, Bulgaria

Team : _____ Solver : _____ ID : _____

1. Given a regular hexagon, we construct all possible triangles from its vertices. We find that:

- There are L possible acute triangles.
- There are M possible right triangles.
- There are N possible obtuse triangles.

Triangles are considered distinct if they have at least one different vertex, even if the triangles are congruent. What is the value of $L \times M \times N$?



Answer: _____



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2. Alex, having only one 20-dollar bill, went to buy lunch. While giving him back change, the cashier got distracted and mixed up the dollars and cents—paying dollars instead of cents and cents instead of dollars. Alex put the change in his wallet without looking at it. After a while he decided to buy a slice of bread as an addition to his lunch. He paid 15 cents for it and then noticed that the amount he had with him now was double of what he should have had after paying for the lunch and the slice of bread (if the cashier had not made a mistake). How many dollars and how many cents is the correct change that Alex should have received after buying lunch? (Note: 1 dollar = 100 cents)

Answer: _____ dollars and _____ cents



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3. Two couriers, Alice and Bob, travel at different constant speeds. Alice departs from village A towards village B and Bob departs from village B towards village A at the same time. When they meet at point C , both turn around and return to their starting points. After some time, Alice realizes that she has forgotten to give Bob a package, so she turns around again and catches up with Bob halfway between C and B . After delivering the package, Alice turns around to go back to A and Bob continues to B . If the distance between A and B is 24 km, when Bob arrives at B , what is the distance, in km, of Alice from A ?

Answer: _____ km



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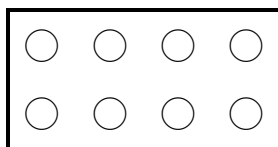
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4. There are 8 circles on a rectangular board as shown in the diagram below. The distances between any pair of adjacent circles, horizontally or vertically, are all equal. Boris wants to colour some circles in red. How many ways can Boris colour some of the circles so that the resulting board has no line of symmetry?



Answer: _____ ways



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5. What is the smallest positive integer n such that n^2 ends with 9009?

Answer: _____



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6. The number 3 has the following properties: it is one less than a square number, and five times of it is also one less than a square number.

$$3 = 4 - 1 = 2^2 - 1$$

$$5 \times 3 = 16 - 1 = 4^2 - 1$$

24 is another number with the same properties:

$$24 = 25 - 1 = 5^2 - 1$$

$$5 \times 24 = 121 - 1 = 11^2 - 1$$

What is the smallest number with the same properties that is greater than 25?

Answer: _____



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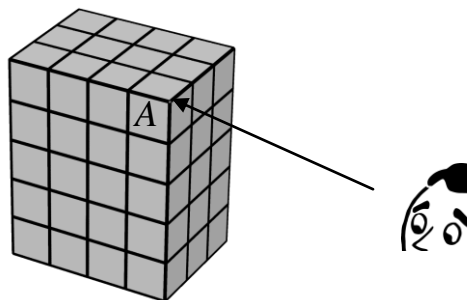
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7. An $a \times b \times c$ rectangular block with $a \leq b \leq c \leq 20$ is made from unit cubes. When it is viewed from one vertex, such that the three faces intersecting at this vertex can be entirely seen, exactly 487 cubes are visible. What is the number of the unit cubes not visible? List down all possible answers.

For example, the diagram on the right is a $3 \times 4 \times 5$ rectangular block. When it is viewed from vertex A, 36 cubes are visible.



Answer: _____ unit cubes



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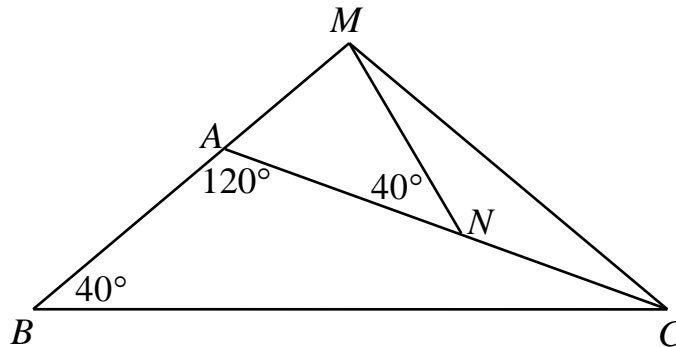
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8. In triangle ABC , $\angle BAC = 120^\circ$ and $\angle ABC = 40^\circ$. Let N be a point on AC such that $AN = AB$. Let M be a point on the line BA such that $\angle ANM = 40^\circ$, as shown in the diagram below. What is the measure, in degrees, of $\angle BMC$?



○

Answer: _____



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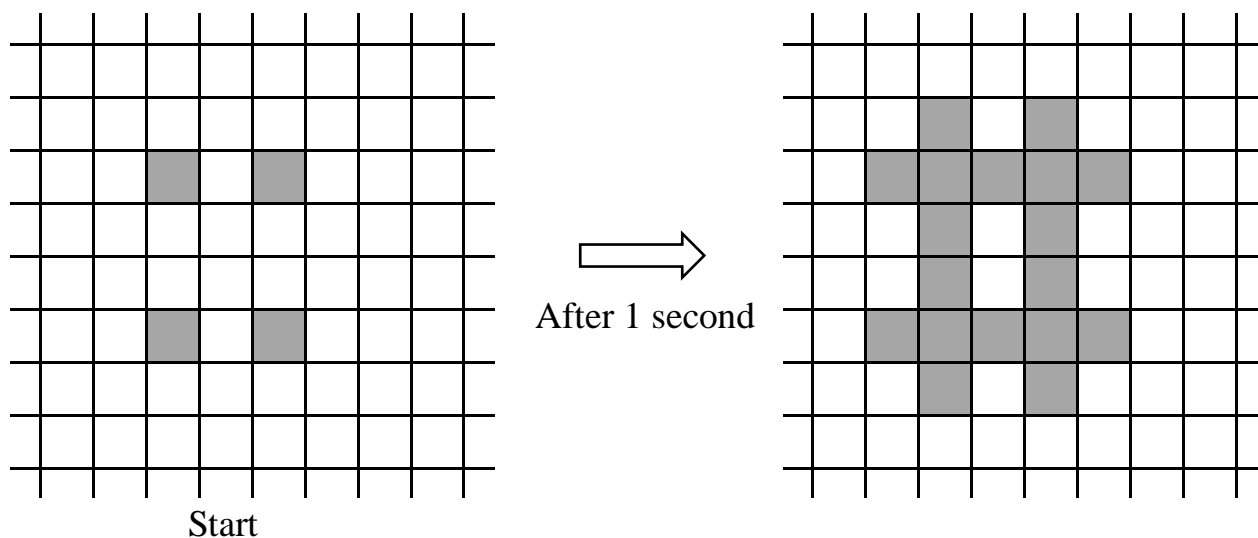
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9. A wizard paints in grey four squares of an infinite grid table, as shown in the diagram below on the left. Then he casts a spell so that, at every second, each square that has at least one common side with those grey squares becomes also grey. For example, after 1 second, there will be 18 grey squares as shown in the diagram below on the right. What is the number of grey squares after 60 seconds?



Answer: _____ grey squares



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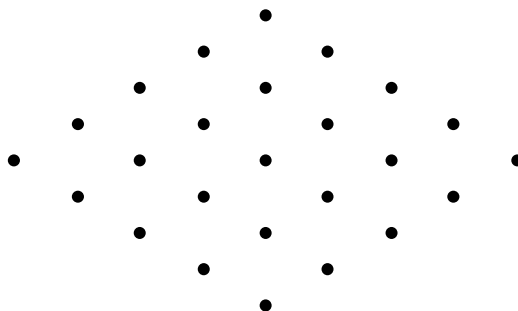
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10. The rhombus shape below is formed by using 25 dots arranged in a triangular grid. The distances between any two adjacent dots are equal. What is the number of equilateral triangles that can be drawn with 3 dots from these 25 dots as vertices?



Answer: _____ equilateral triangles