Bulgaria International Mathematics Competition 2023 (Virtual)

Bulgaria, 1st to 7th July 2023

Invitational World Youth Mathematics Intercity Competition

Team Contest

Time limit: 70 minutes

Information:

- You are allowed 70 minutes for this paper, consisting of 10 questions printed on separate sheets. For questions 1, 3, 5, 7 and 9, only numerical answers are required. For questions 2, 4, 6, 8 and 10, full solutions are required.
- Each question is worth 40 points. For odd-numbered questions, no partial credit is given. There are no penalties for incorrect answers, but you must not give more than the number of answers being asked for. For questions asking for several answers, full credit will only be given if all correct answers are found. For even-numbered questions, partial credit may be awarded.
- Diagrams shown may not be drawn to scale.

Instructions:

- Key in your name, your contestant ID and your team's name before going to next page. Write down your team's name in the space of every question sheet.
- You may not use instruments such as protractors, calculators and electronic devices.
- For the odd-numbered questions, Enter your answers in the column provided after each question. For all answers, **no need to key in their units**. The format as following:
  1. For decimal *a*.*bc*, where *a*,*b* and *c* are digits, key in *a*.*bc*.
  2. For fraction $\frac{a}{b}$, where *a* and *b* are coprime, key in *a*/*b* (For example, if your answer is $3\frac{2}{5}$, please key in **17/5**).
  3. For ratio $a : b$, key in *a*:*b* or *a*;*b* (no need space after ":" or ";").
  4. For number pair (*a*, *b*, *c*, …), key in *a*,*b*,*c*,….(no need space after ",").
  5. If the solution is $a+b\times\sqrt{c}$, please key in a,b,c. (no need space after ",". For example, **if your answer is** $3+\sqrt{5}$**, please key in 3,1,5**).
- For even-numbered questions, Write your full solution on the blank papers provided.
- During the first 10 minutes, the four team members examine the first 8 questions together, and altogether discuss them. Then they distribute the questions among themselves, with each team member is allotted at least 1 question.
- During the next 35 minutes, the four team members write down the solutions of their allotted problems, with no further communication / discussion among themselves.
- During the last 25 minutes, the four team members work together to solve the last 2 questions.
- At the end of each part contest, you must click "send" for the odd-numbered questions and scan or take a photo of your even-numbered questions solutions then upload to the given website.

English Version

Team: ______________________________

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3rd July, 2023, Bulgaria

Team：_____________ Solver：_____________ ID：____________

1. The equation $x^3-9x^2+11x-1=0$ has the non-negative real roots p, q and r. If $s=\sqrt{p}+\sqrt{q}+\sqrt{r}$, what is the value of $|s^4-18s^2-8s|$?

Answer: __________________________

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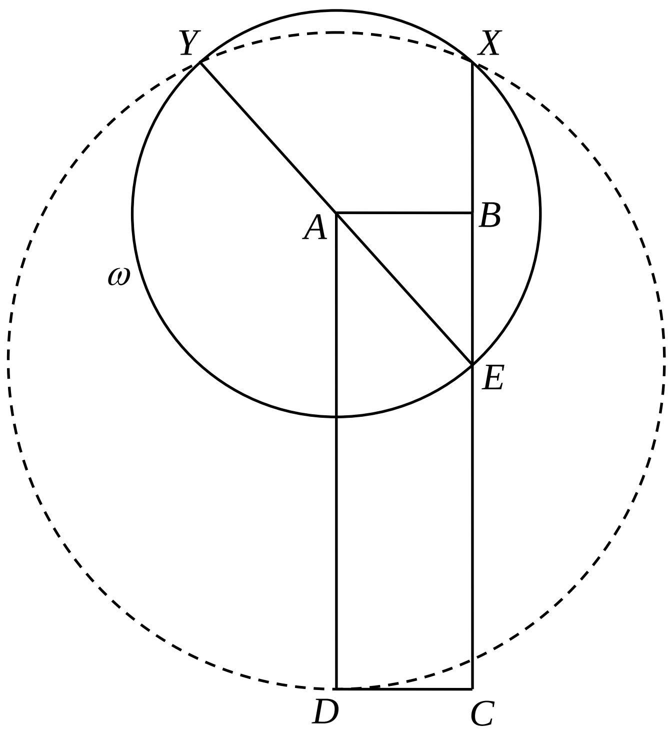
2. Let $ABCD$ be a rectangle, E is a point on the line segment BC. The circle ω has center A and passes through E. Extension lines EB and AE meet ω at X and Y, respectively, as shown in the diagram below. Prove that DC is tangent to the circle passing through Y, X and D.



Team Contest

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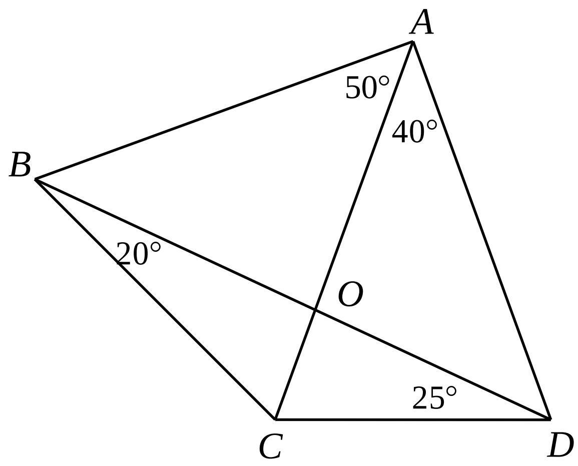
3. The convex quadrilateral *ABCD* has $\angle CAD = 40°$, $\angle BAC = 50°$, $\angle CBD = 20°$ and $\angle CDB = 25°$, as shown in the diagram below. If *O* is the intersection of *AC* and *BD*, what is the measure, in degrees, of $\angle AOB$?



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Answer: ________________

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3rd July, 2023, Bulgaria

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4. Dinko placed some marbles into one hundred black bags, which are numbered from 1 to 100. Each time, you are allowed to ask Dinko about the parity of the total number of all the marbles in any 15 distinct bags and Dinko will give you the correct answer. What is the least number of times you need to ask Dinko, in order to determine the parity of the number of the marbles in bag 1 correctly? Show an example and prove your answer.

Answer: __________ times

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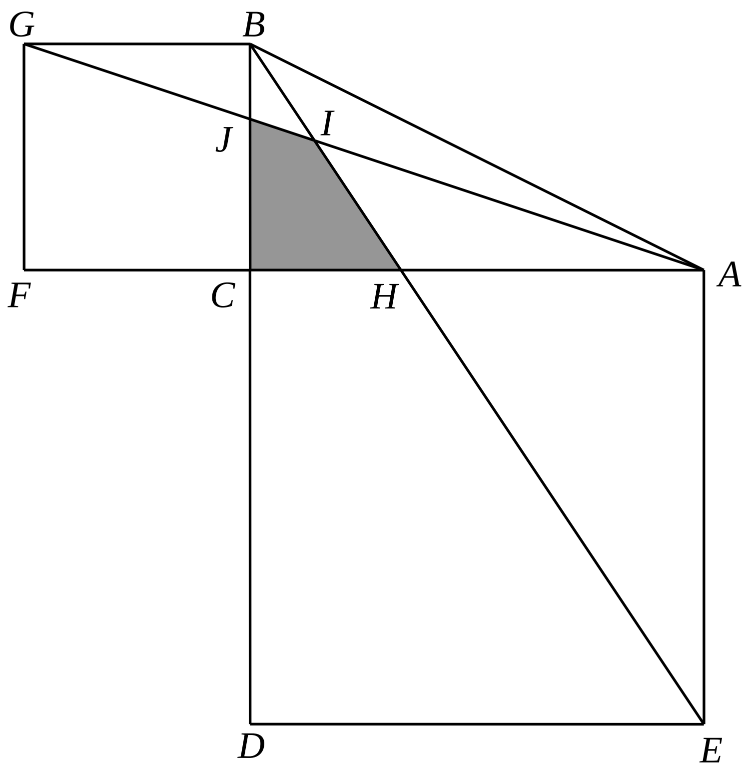
5. Let ABC be a triangle with $\angle C = 90°$, $AC = 28$ cm and $BC = 14$ cm. Squares $ACDE$ and $BCFG$ are constructed outside the triangle ABC. Let BE intersect AC and AG at H and I, respectively. AG intersects BC at J, as shown in the diagram below. What is the area, in cm^2, of quadrilateral $CHIJ$?



Answer: ______________________ cm^2

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6. There exists an infinite arithmetic progression satisfying the following properties:
   - Each term of the progression is a positive integer.
   - The common difference of the progression is *d*.
   - No term of the progression appears in the Fibonacci sequence.

   What is the smallest positive integer *d* for which this is possible?
   (The Fibonacci sequence is defined as $F_1=1$, $F_2=1$, $F_{n+2}=F_n+F_{n+1}$ for $n\geq 1$)

Answer: ________________________

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7. A perfect square N is called a "*BIMC number*" if there exist three positive integers a, b and c, where $a<b<c$, such that

$$N=(b-a)(c-a)(c-b)=ab+ac+bc.$$

For example, 230400 is a BIMC number since $230400=480\times 480$ and $230400=(288-224)(324-224)(324-288)=224\times 288+224\times 324+288\times 324$.

What is the smallest BIMC number?

Answer: ________________

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8. Tim wants to place 1600 points inside a unit square such that there is at least one point inside every rectangle of area $\frac{1}{200}$ and with sides parallel to those of the square. Is it possible to do it? If your answer is yes, please give an example and show that it works. If your answer is no, please prove it is impossible.

Answer: ____________________

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9. How many 7-digit positive integers with distinct digits are **not** divisible by 9?

Answer: ________________

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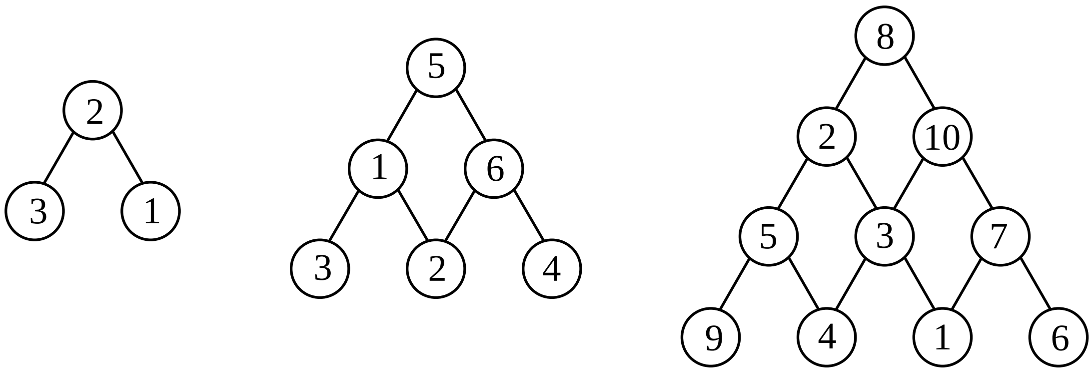
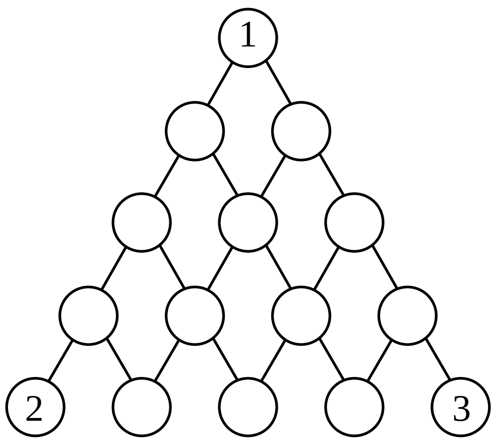
10. A triangular arrangement of numbers is called a *smart triangle of order n* if one can arrange all the positive integers from 1 to $\frac{n(n+1)}{2}$ in a triangular structure such that, apart from the bottom row of n numbers, each number in a subsequent row above it is the absolute difference or the sum of the two numbers immediately below it.
The triangular structures represented below, from left to right, are some examples of smart triangles of order 2, 3 and 4, respectively.



(a) Does there exist a smart triangle of order 5, with the numbers 1, 2 and 3 in the vertices, as shown in the diagram below? If your answer is yes, please give an example. If your answer is no, please prove it is impossible. (10 marks)



(b) Does there exist a smart triangle of order 6? If your answer is yes, please give an example. If your answer is no, please prove it is impossible. (30 marks)

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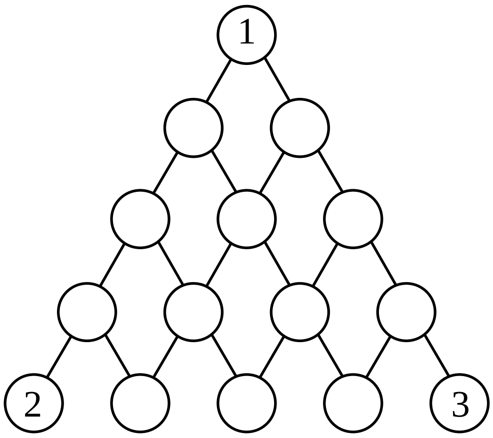
10. Answer:

(a) ☐ My answer is **yes** and the example is shown on the right.

☐ My answer is **no** and the proof is:



(b) ☐ My answer is **yes** and the example is shown on the right.

☐ My answer is **no** and the proof is: