



India International Mathematics Competition 2024

Lucknow, 26th to 31st July 2024

Elementary Mathematics International Contest

Team Contest

Time limit: 70 minutes

Information:

- You are allowed 70 minutes for this paper, consisting of 10 questions printed on separate sheets. For questions 1, 3, 5, 7 and 9, only numerical answers are required. For questions 2, 4, 6, 8 and 10, full solutions are required.
- Each question is worth 40 points. For odd-numbered questions, no partial credits are given. There are no penalties for incorrect answers, but you must not give more than the number of answers being asked for. For questions asking for several answers, full credit will only be given if all correct answers are found. For even-numbered questions, partial credits may be awarded.
- Diagrams shown may not be drawn to scale.

Instructions:

- Write down your team's name in the space provided on every question sheet.
- Enter your answers at the bottom right corners of the corresponding question sheets.
- During the first 10 minutes, the four team members examine the first 8 questions together and discuss them. After 10 minutes they distribute the questions among themselves, with every team member allotted at least 1 question.
- During the next 35 minutes, the four team members write down the solutions of their allotted problems on the respective question sheets, with no further communication or discussion.
- During the last 25 minutes, the four team members work together to write down the solutions of the last 2 questions on the respective question sheets.
- It is forbidden to use instruments such as protractors, calculators and electronic devices.
- At the end of the contest, you must hand in the envelope containing all question sheets and all scrap paper.

English Version

Team: _____



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Elementary Mathematics International Contest **Team Contest**

28th July, 2024, Lucknow, India

Team : _____ Solver : _____ ID : _____

1. The save rate of a goalkeeper is the percentage given by the number of shots on his target that he has saved divided by the total number of shots on his target. At a given time during a football match, a certain goalkeeper's save rate was 75%. After failing to save the next three shots on his target, his save rate went down to 60%. What is the minimum number of shots on target that he needs to save in order to raise his save rate back to at least 75%?

Answer: _____ shots



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2. The following number:
“123456789101112131415.....444424444344444”
is made up by writing all the integers from 1 to 44444 consecutively in
increasing order from left to right.
How many times does the string “2024” appear in the given number?

Answer: _____ times



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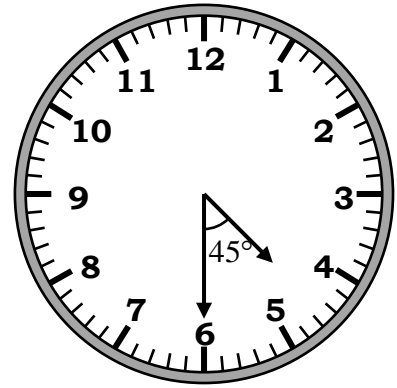
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3. How many moments in a day (from 00:00 to 23:59) will the hour and the minute hands of an analog clock (12 hours system) form an angle of $15k$ degrees, where k is an integer so that $0 \leq k < 24$? (The diagram shown is one moment at 04:30, having an angle of $15^\circ \times 3 = 45^\circ$)



Answer: _____ moments



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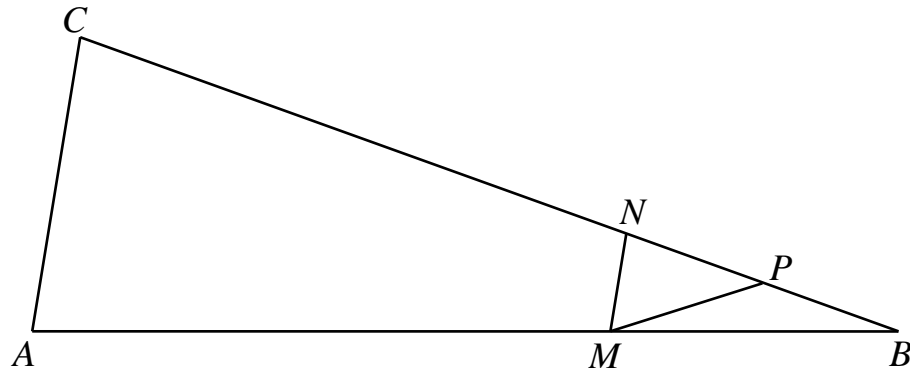
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4. In the diagram below, ABC is a triangle, where point M lies on the side AB , point N lies on the side BC and $MN \parallel AC$. It is also known that P is the midpoint of BN , and that $BN : PC = 2 : 5$. If the area of the triangle MBP is 1 cm^2 , what is the area, in cm^2 , of triangle ABC ?



Answer: _____ cm^2



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5. The column 1 to column 7 of the table below will continue downward, from left to right and are being filled with the positive integers starting from “1”, written in increasing order and skipping all multiples of 5 or 7.

	column 1	column 2	column 3	column 4	column 5	column 6	column 7
row 1	1	2	3	4	6	8	9
row 2	11	12	13	16	17	18	19
row 3	22	23	24	26	27	29	31
row 4	32	33	34	36	37	38	39
.....

If 2024 is located in row x and column y , what is the value of $x + y$?

Answer: _____



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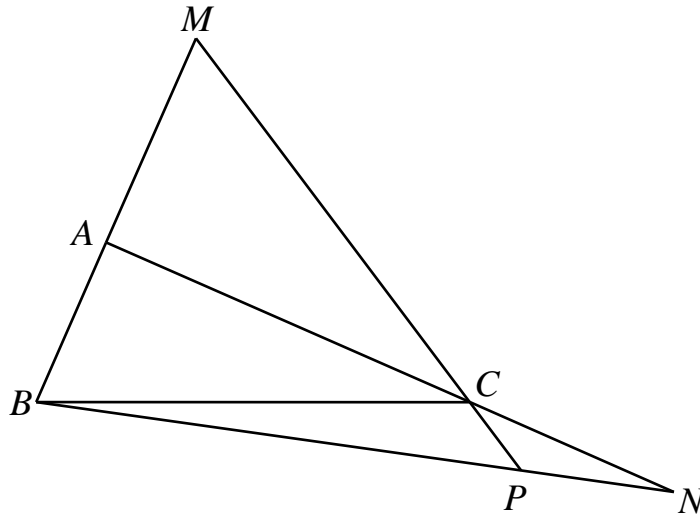
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6. In the diagram below, triangle ABC is a right triangle at A with $AC > AB$. Let point M be on ray BA such that $BM = AC$. Let point N be outside triangle ABC and on the ray AC such that $CN = AM$. If ray MC intersects BN at P , what is the measure, in degrees, of $\angle MPB$?



o

Answer: _____



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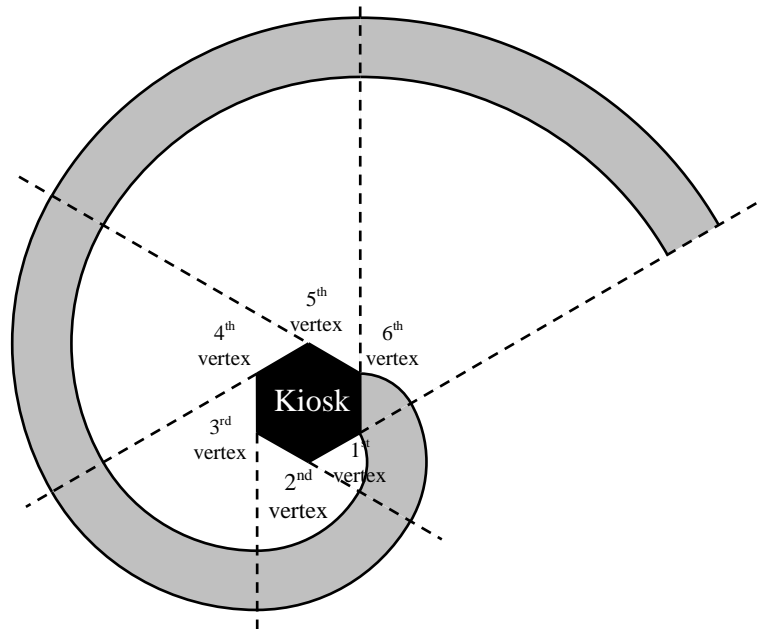
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7. There is a regular hexagon-shaped kiosk in the middle of a garden. To get to the kiosk, there's a spiral path that leads right up to one of the sides of the hexagon. To create the path, the builder used an interesting technique. He tied a rope counterclockwise around the kiosk and used a pencil attached to the end of the rope to draw the outer edge of the path starting from one vertex of the hexagon. After that, he untangled the rope clockwise to draw the inside border. The same process was repeated, starting from the next vertex of the hexagon, as shown in the diagram below. What is the total area (shaded area in the diagram not including the kiosk), in m^2 , of the path to the kiosk if the kiosk measures 6 meters on each side? (Take $\pi = \frac{22}{7}$)



Answer: _____ m^2



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8. A special calculator has three commands (A, B and C) that can be used to modify the number that is displayed on the calculator.

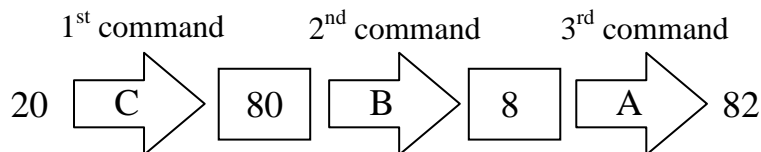
The three commands are as follows:

Command A: Insert 2 as the rightmost digit of the number.

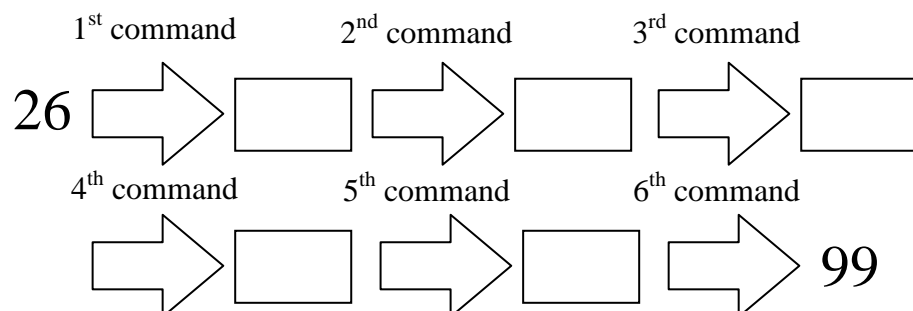
Command B: Replace the number with the sum of all its digits.

Command C: Replace each digit d of the number with the digit $10 - d$, except that 0 is replaced with 0. (For example, 1085 would become 9025.)

For example, this is how one can arrive at 82 from 20 in 3 steps



If the calculator initially displays the number 26, show the step by step process on how to get the calculator to display 99 by using exactly six commands.



Answer: _____



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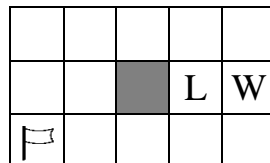
Team : _____ Solver : _____ ID : _____

9. Ann and Bob are playing a game where they take turns moving a counter on a grid. Ann goes first. The goal is to move the counter to the bottom-left corner. In a single move, a player must perform exactly one of the following two actions:

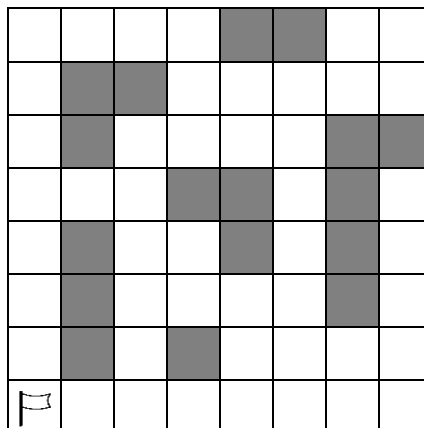
- Move the counter one or more cells to the left.
- Move the counter one or more cells down.

However, the counter **CANNOT** be moved onto or across a shaded cell.

Ann has a “winning cell” if, when the game starts with the counter on that cell and Ann and Bob both play optimally, Ann has a winning strategy. Otherwise, the cell is known as a “losing cell”. For example, in the diagram below, cell L is a losing cell for Ann since her only option is to move the counter down 1 cell, after which Bob can win by moving the counter 3 cells to the left. Cell W is a winning cell for Ann because Ann can move the counter 1 cell to the left, causing Bob to move the counter down 1 cell. Ann can then win by moving the counter 3 cells to the left.



In the 8×8 grid below, how many winning cells does Ann have?



Answer: _____ winning cells



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10. Each square of an infinite grid has been filled with exactly one digit. Elsa places an O-tetromino on this grid covering four squares along the grid lines:

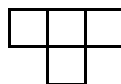


The minimum number of different digits used to fill the infinite grid so that the four squares will have different digits, regardless the position of the O-tetromino, is 4. The grid on the right is an example of such filling.

1	2	1	2	1	2	1	2
3	4	3	4	3	4	3	4
1	2	1	2	1	2	1	2
3	4	3	4	3	4	3	4
1	2	1	2	1	2	1	2
3	4	3	4	3	4	3	4
1	2	1	2	1	2	1	2
3	4	3	4	3	4	3	4

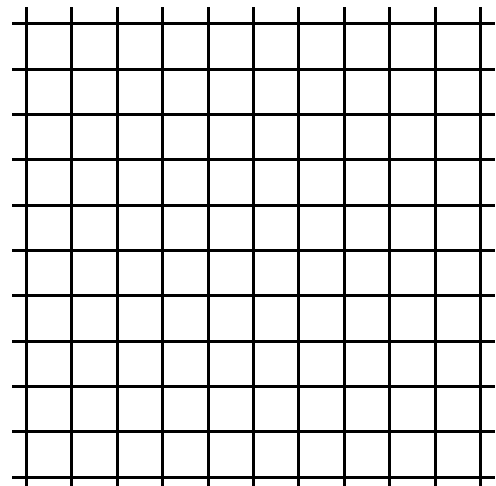
When Elsa places a T-tetromino on this grid covering four squares along the grid lines, what is the minimum number n of different digits that must be used for the infinite grid so that the four squares covered by the tetromino will always have different digits?

The T-tetromino can be flipped or rotated.



T-tetromino

(Note: You must explain why using $n - 1$ different digits is insufficient and give a filling example to show that using n digits is sufficient.)



Answer: $n =$ _____, filling way is _____