India International Mathematics Competition 2024

Lucknow, 26th to 31st July 2024

Invitational World Youth Mathematics Intercity Competition

Team Contest

Time limit: 70 minutes

Information:

- You are allowed 70 minutes for this paper, consisting of 10 questions printed on separate sheets. For questions 1, 3, 5, 7 and 9, only numerical answers are required. For questions 2, 4, 6, 8 and 10, full solutions are required.
- Each question is worth 40 points. For odd-numbered questions, no partial credits are given. There are no penalties for incorrect answers, but you must not give more than the number of answers being asked for. For questions asking for several answers, full credit will only be given if all correct answers are found. For even-numbered questions, partial credits may be awarded.
- Diagrams shown may not be drawn to scale.

Instructions:

- Write down your team's name in the space provided on every question sheet.
- Enter your answers at the bottom right corners of the corresponding question sheets.
- During the first 10 minutes, the four team members examine the first 8 questions together and discuss them. After 10 minutes they distribute the questions among themselves, with every team member allotted at least 1 question.
- During the next 35 minutes, the four team members write down the solutions of their allotted problems on the respective question sheets, with no further communication or discussion.
- During the last 25 minutes, the four team members work together to write down the solutions of the last 2 questions on the respective question sheets.
- It is forbidden to use instruments such as protractors, calculators and electronic devices.
- At the end of the contest, you must hand in the envelope containing all question sheets and all scrap paper.

English Version

Team: ______________________________

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Team：____________ Solver：____________ ID：____________

1. Let C, M and S be positive integers that satisfy the equation

$$C^2 + M^2 + 2^{2024} = 25 \times 2^S .$$

Given that $C \le M$, what is the solution (C, M, S) of the equation?

Answer: (, ,)

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2. Let $f(x)=ax^2+bx+c$ be a function, where a, b and c are integers and $a>0$. If x_1 and x_2 are the two roots of $f(x)=0$, where $0<x_1<x_2<1$, what is the minimum possible value of a?

Answer: ____________________________

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3. The set $M = \{1, 2, 3, \ldots, 2024\}$ is to be partitioned into two subsets, E and O, such that the subset E contains the elements of M for which the sum of digits is even, and the subset O contains the elements of M for which the sum of digits is odd. If e is the sum of all the elements in E and o is the sum of all the elements in O, what is the value of $e - o$?

Answer: ____________________

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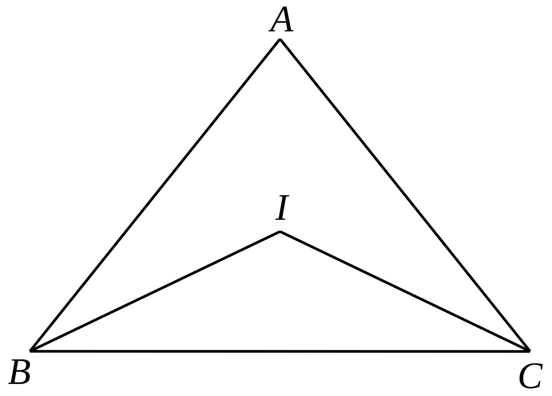
4. In the diagram below, ABC is a triangle, where point I is the intersection of the internal angle bisectors of $\angle ABC$ and $\angle ACB$. If $BI + AC = CI + AB$, show that triangle ABC is isosceles.



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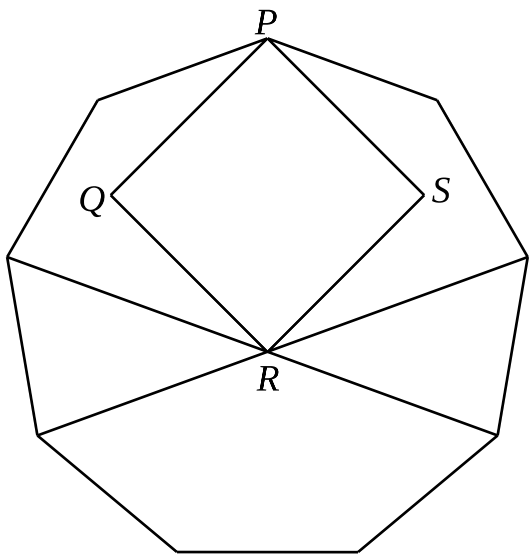
5. In the diagram below, $PQRS$ is a square inside the regular 9-gon and point P is also the vertex of the regular 9-gon. If the side length of the 9-gon is 48 cm, then what is the area, in cm^2, of square $PQRS$?



Answer: __________ cm^2

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6. Let $\overline{ABCD}$ be a four-digit number. If the last two digits of $\overline{ABCD}^{2025}$ are 57, what is the largest possible value of the two-digit number $\overline{CD}$?

Answer: __________________________

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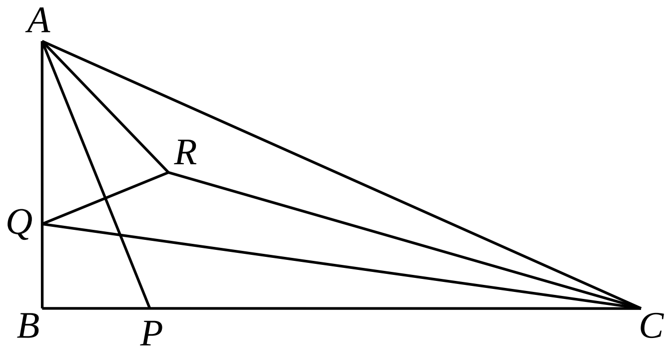
7. In the diagram below, triangle ABC is right-angled at B. Let points P and Q be on BC and AB, respectively, such that $\angle PAB=\frac{1}{3}\angle CAB$ and $\angle QCB=\frac{1}{3}\angle ACB$. If point R is inside triangle ABC such that $\angle RAC=\angle BAP$, $\angle RCA=\angle BCQ$ and $\angle QRC$ is an integer value in degrees, what is the minimum possible value, in degrees, of $\angle QRC$?



Answer: ____________________ $^\circ$

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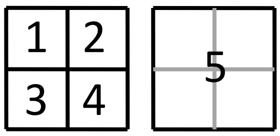
8. A *subsquare* of a grid of unit squares is a square of any size that is bounded by lines of the grid. For example, a 2×2 grid has 5 *subsquares*.



Suppose a $m\times n$ grid of unit squares has exactly 2024 *subsquares* of all sizes, what is the smallest possible value of mn?

Answer: ________________________

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9. There are n donkeys standing along the circumference of a circular farm playing a game described as follows. The donkeys are numbered 1, 2, 3, ..., n in the clockwise direction. Starting from the donkey numbered 1 and proceeding clockwise, the donkeys take turns yelling the letters “D”, “O”, “N”, “K”, “E”, “Y”, “D”, “O”, “N”, “K”, “E”, “Y”, “D”, “O”…. Whenever a donkey yells the letter “Y”, it leaves the circle before the next donkey yells “D”. This process continues until there is one donkey left in the circle, and this remaining donkey wins the game.

For example, if $n=4$, then the game proceeds as follows:

- Donkey 1 yells “D”, 2 yells “O”, 3 yells “N”, 4 yells “K”, 1 yells “E”, 2 yells “Y” and leaves the circle.
- Donkey 3 yells “D”, 4 yells “O”, 1 yells “N”, 3 yells “K”, 4 yells “E”, 1 yells “Y” and leaves the circle.
- Donkey 3 yells “D”, 4 yells “O”, 3 yells “N”, 4 yells “K”, 3 yells “E”, 4 yells “Y” and leaves the circle.

Donkey 3 is the last remaining donkey in the circle, thus it wins the game.

It is given that when $n=186$, then the winning donkey’s number is 10. What is the smallest value of n, where $n>186$, such that the winning donkey’s number is 100?

Answer: ____________________

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10. Captain Crook and his pirates have salvaged 10 golden bricks from Golden Island. Now, Captain Crook and his crew will play a game on how to divide the loots within them. The crew prepares several chests, each containing at most one golden brick (and everyone can see the inside of the chests). The crew then chains together the chests so that the chests and the chains form a tree. To play the game, the captain and the crew will alternately take chests, with the captain taking first, following this rule: whenever possible, one has to take a chest that was directly chained to a chest that was taken already. If not possible, one can take any chest. (In particular, the captain can start with any chest he likes because none have been taken.) For example, suppose that there were only 3 golden bricks and the crew chains the chests like this:

```
G —— 0 —— G —— 0 —— G
```

where G represents a chest containing a golden brick, and 0 represents an empty chest. Then the captain will take the left-most G, forcing the crew to take the 0, then captain G, crew 0, and finally captain G.

Let's take another example. Suppose that the crew chains the chests like this:

```
G —— 0 —— G —— 0
          |
          G
```

The captain will guarantee two golden bricks by starting with the leftmost G or the G between the 0s.

Finally, if the crew chains like this:

```
G —— 0 —— 0 —— G
     |
     0 —— G
```

then the captain will get only one golden brick.

Is it possible for the crew to chain the chests in a certain way so that Captain Crook will get at most one brick? If not, how should the crew minimize the number of bricks Captain Crook can get? Explain your strategy in full detail.

(Write your answer on a separate paper)