Vietnam International Mathematics Competition 2025

Danang City, 14^{th} to 19^{th} August 2025

Invitational World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes

Information:

- You are allowed 120 minutes for this paper, consisting of 12 questions in Section A to which only numerical answers are required, and 3 questions in Section B to which full solutions are required.
- Each question in Section A is worth 5 points. No partial credits are given. There are no penalties for incorrect answers, but you must not give more than the number of answers being asked for. For questions asking for several answers, full credit will only be given if all correct answers are found. Each question in Section B is worth 20 points. Partial credits may be awarded.
- Diagrams shown may not be drawn to scale.

Instructions:

- Write down your name, your contestant ID and your team's name on the answer sheet of Section A and on **each page of Section B**.
- For Section A, write your answers in the space provided on the answer sheet.
  For Section B, write down your solutions on spaces provided after individual questions.
- You are allowed to use HB, B, and 2B pencils and ball-point pens with black or blue ink.
- You cannot use instruments such as protractors, calculators and electronic devices, smart watches.
- At the end of the contest, you must hand in the envelope containing the question sheets, the answer sheet and all scrap paper.

English Version

Section A.

In this section, there are 12 questions. Fill in the correct answer in the answer sheet of each question. Each correct answer is worth 5 points.

1. An ordered triple of positive integers (a, b, c) is called a *V-triple* if $a<b<c$ and it satisfies all the following conditions:

- a is a factor of $b+c$;
- b is a factor of $c+a$;
- c is a factor of $a+b$;
- The sum of a, b and c is less than or equal to 2025.

How many *V-triples* are there in total?

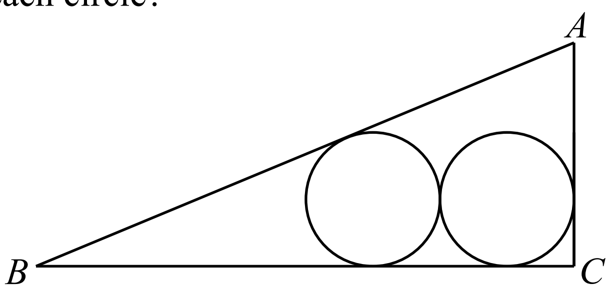
2. Let $P_1P_2...P_n$ be a regular n-gon, where $n \geq 29$. If $\angle P_5P_{27}P_{29} = 165°$, what is the value of n?

3. Let x, y and z be positive integers such that

$$\begin{cases} z^3 - x^3 - y^3 = 3xyz \\ x^2 + y^2 + z^2 \leq 150 \end{cases}$$

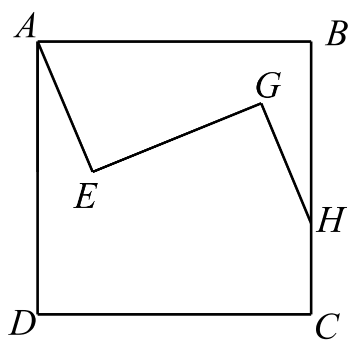
What is the maximum possible value of $x+y+z$?

4. In the diagram below, ABC is a triangle with $AB = 13$ cm, $BC = 12$ cm and $AC = 5$ cm. Inside this triangle, we consider two equal circles, that are externally tangent to each other, such that one of them is tangent to sides AB and BC, and the other one is tangent to sides BC and AC. What is the length, in cm, of the radius of each circle?



5. What is the sum of all two-digit positive integers $\overline{ab}$, such that $\overline{ab}$ divides the four-digit number $\overline{baba}$?

6. In the diagram below, $ABCD$ is a square. Let E and G be two points inside square $ABCD$ and H be a point on BC such that $\angle AEG = \angle EGH = 90°$ and $EG = BH$. If $AE = 13$ cm and $GH = 12$ cm, what is the side length, in cm, of square $ABCD$?



7. Anh and Bao are playing a game using the 2025 calendar. First, Anh chooses a date in January. Next, Bao chooses a date later in the calendar year, but he must keep either the month or the day from Anh's previous move. The game continues in a similar way where they take turns and whoever chooses December 31st is the winner. One example of a played game is shared below, with Bao winning:

Anh :	January 7th
Bao :	April **7th**
Anh :	May **7th**
Bao :	**May** 19th
Anh :	**May** 21st
Bao :	September **21st**
Anh :	November **21st**
Bao :	**November** 30th
Anh :	December **30th**
Bao :	**December** 31st

If Anh has a winning strategy, which date in January should he choose on the first move?

8. What is the number of ordered pairs of positive integers (x, y) such that $\frac{1}{x}+\frac{1}{y}=\frac{1}{2025}$?

9. What is the sum of all positive integers n such that $n!$ has exactly 100 trailing zeros?

10. Consider a convex 2025-gon. We draw a subset of its diagonals such that each drawn diagonal (excluding the two endpoints) intersects exactly one of the other drawn diagonals. What is the maximum possible number of such diagonals that can be drawn?

11. What is the sum of all prime numbers p such that p^2 divides $11^{p^2}+1$?

12. What is the largest real number k such that

$$(x^2+y^2+4x-4y-1)^2+(x^2+y^2-12x-16y-k)^2=0$$

for some ordered pair of real numbers (x, y)?

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Team:__________________ ***Name***: _________________ ***ID.***: ______________

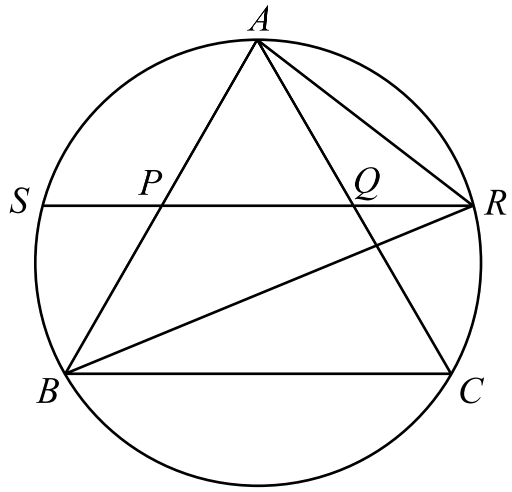
Section B.

Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. Let n be any integer where $n \ge 2$. Now, consider all the fractions in the form $\frac{1}{pq}$, where gcd $(p, q) = 1$, $0 < p < q \le n$ and $p + q > n$. Prove that the sum of all such fractions is $\frac{1}{2}$.

Team:____________________ **Name**: ____________________ **ID.**: ________________

2. In the diagram below, circle O is the circumcircle of the equilateral triangle ABC. Points P and Q are the midpoints of AB and AC, respectively. Line PQ intersects circle O at points S and R. If $AR = 10$ cm, what is the length, in cm, of BR?



Answer: ________________________ cm

Team:____________________ ***Name***: ____________________ ***ID.***: ________________

3. Alex has a fair six-faced die with the fractions $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}$, $\frac{9}{2}$ and $\frac{11}{2}$ on its faces. Ben, on the other hand, has a spinning wheel, that is divided into n equal parts. An integer from 0 to $n-1$ is written on each part, with each number appearing exactly once.

Alex rolls his die, and Ben spins his wheel at the same time. Let p be the probability that Ben obtains a greater number than the one Alex obtains. What is the smallest value of n for which p is at least $\frac{1}{2}$?

Answer: ____________________________