Taiwan International Mathematics Competition 2012 (TAIMC 2012)
World Conference on the Mathematically Gifted Students ---- the Role of Educators and Parents
Taipei, Taiwan, 23rd~28th July 2012

Invitational World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes

Instructions:

- Do not turn to the first page until you are told to do so.
- Remember to write down your team name, your name and contestant number in the spaces indicated on the first page.
- The Individual Contest is composed of two sections with a total of 120 points.
- Section A consists of 12 questions in which blanks are to be filled in and only **ARABIC NUMERAL** answers are required. For problems involving more than one answer, points are given only when **ALL** answers are correct. Each question is worth 5 points. There is no penalty for a wrong answer.
- Section B consists of 3 problems of a computational nature, and the solutions should include detailed explanations. Each problem is worth 20 points, and partial credit may be awarded.
- You have a total of 120 minutes to complete the competition.
- No calculator, calculating device, watches or electronic devices are allowed.
- Answers must be in pencil or in blue or black ball point pen.
- All papers shall be collected at the end of this test.

English Version

Team:_____________ Name:__________________ No.: _____________ Score:_________

For Juries Use Only

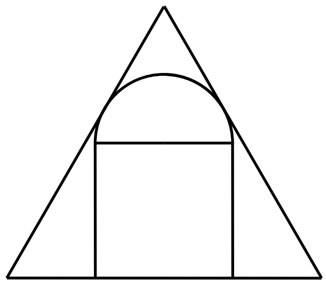
No.	Section A												Section B			Total	Sign by Jury
	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3		
Score																	
Score																	

Section A.
In this section, there are 12 questions. Fill in the correct answer in the space provided at the end of each question. Each correct answer is worth 5 points.

1. Determine the maximum value of the difference of two positive integers whose sum is 2034 and whose product is a multiple of 2034.

Answer : _________

2. The diagram below shows a semicircle sitting on top of a square and tangent to two sides of an equilateral triangle whose base coincides with that of the square. If the length of each side of the equilateral triangle is 12 cm, what is the radius of the semicircle, in cm?



Answer : _______ cm

3. A four-digit number $\overline{abcd}$ is a multiple of 11, with $b + c = a$ and the two-digit number $\overline{bc}$ a square number. Find the number $\overline{abcd}$.

Answer : _________

4. The area of the equilateral triangle ABC is $8+4\sqrt{3}\,\text{cm}^2$. M is the midpoint of BC. The bisector of $\angle MAB$ intersects BM at a point N. What is the area of triangle ABN, in cm^2?

Answer: _______ cm^2

5. There is a 2×6 hole on a wall. It is to be filled in using 1×1 tiles which may be red, white or blue. No two tiles of the same colour may share a common side. Determine the number of all possible ways of filling the hole.

Answer : _________

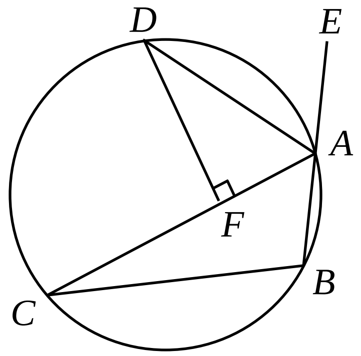
6. Let $N = 1^9 \times 2^8 \times 3^7 \times 4^6 \times 5^5 \times 6^4 \times 7^3 \times 8^2 \times 9^1$. How many perfect squares divide N?

Answer : _________

7. How many positive integers not greater than 20112012 use only the digits 0, 1 or 2?

Answer : _________

8. The diagram below shows four points *A*, *B*, *C* and *D* on a circle. *E* is a point on the extension of *BA* and *AD* is the bisector of $\angle CAE$. *F* is the point on *AC* such that *DF* is perpendicular to *AC*. If $BA = AF = 2$ cm, determine the length of *AC*, in cm.



Answer : ________ cm

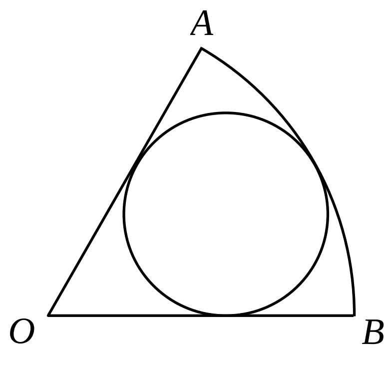
9. There are 256 different four-digit numbers $\overline{abcd}$ where each of *a*, *b*, *c* and *d* is 1, 2, 3 or 4. For how many of these numbers will $ad - bc$ be even?

Answer : __________

10. In a plane, given 24 evenly spaced points on a circle, how many equilateral triangles have at least two vertices among the given points?

Answer : __________

11. The diagram below shows a circular sector *OAB* which is one-sixth of a circle, and a circle which is tangent to *OA*, *OB* and the arc *AB*. What fraction of the area of the circular sector *OAB* is the area of this circle?



Answer : __________

12. An 8×8 chessboard is hung up on the wall as a target, and three identical darts are thrown in its direction. In how many different ways can each dart hit the center of a different square such that any two of these three squares share at least one common vertex?

Answer : __________

Section B.
Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. What is the integral part of M, if

$$M=\sqrt{2012\times\sqrt{2013\times\sqrt{2014\times\sqrt{\cdots\sqrt{(2012^2-3)\times\sqrt{(2012^2-2)\times\sqrt{(2012^2-1)\times\sqrt{2012^2}}}}}}}}\ ?$$

Answer：__________

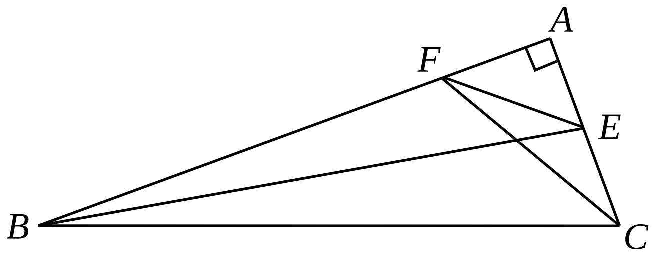
2. Let m and n be positive integers such that

$$n^2<8m<n^2+60(\sqrt{n+1}-\sqrt{n}).$$

Determine the maximum possible value of n.

Answer：__________

3. Let ABC be a triangle with $\angle A = 90^\circ$ and $\angle B = 20^\circ$. Let E and F be points on AC and AB respectively such that $\angle ABE = 10^\circ$ and $\angle ACF = 30^\circ$. Determine $\angle CFE$.



$^\circ$

Answer : ___________