BIMC
BULGARIA INTERNATIONAL MATHEMATICS COMPETITION
Burgas, Bulgaria, 30th June ~ 5th July 2013

BIMC 2013
Elementary Mathematics International Contest

BURGAS MUNICIPALITY

BURGAS
CANDIDATE FOR 2019 EUROPEAN CAPITAL OF CULTURE

Individual Contest

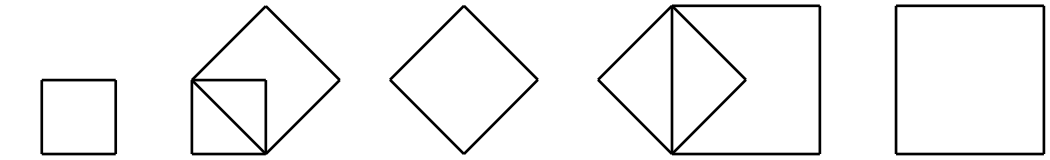
Time limit: 90 minutes

Instructions:

- Do not turn to the first page until you are told to do so.
- Write down your name, your contestant number and your team's name on the answer sheet.
- Write down all answers on the answer sheet. Only Arabic NUMERICAL answers are needed.
- Answer all 15 problems. Each problem is worth 10 points and the total is 150 points. For problems involving more than one answer, full credit will be given only if ALL answers are correct, no partial credit will be given. There is no penalty for a wrong answer.
- Diagrams shown may not be drawn to scale.
- No calculator or calculating device is allowed.
- Answer the problems with pencil, blue or black ball pen.
- All papers shall be collected at the end of this test.

English Version

1. In a sequence of squares, the 1-st one has side length 1 cm. The side length of each subsequent square is equal to the length of a diagonal of the preceding square. The diagram below illustrates the construction of the 2-nd and 3-rd squares. What is the side length, in cm, of the 11-th square?



1-st Square 2-nd Square 3-rd Square

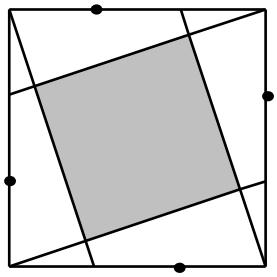
2. Twenty girls stood in a row facing right. Four boys joined the row, but facing left. Each boy counted the number of girls in front of him. The numbers were 3, 6, 15 and 18 respectively. Each girl also counted the number of boys in front of her. What was the sum of the numbers counted by the girls?

3. The diagram below on the left shows ten advertisements A, B, C, D, E, F, G, H, I and J in the ten boxes on a 2 × 5 billboard, on a certain day. Each day, the advertisements move from box to box following a fixed pattern. On the day after, they appear as in the diagram below on the right. How many days will it take before all the advertisements return to their starting positions together for the first time?

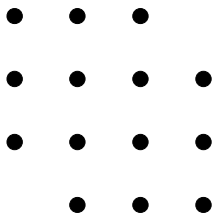
A	B	C	D	E
F	G	H	I	J

G	A	E	F	B
I	C	J	D	H

4. Each side of a square of side length 10 cm is divided into three equal parts. Some of these division points are connected to the vertices of the square, as shown in the diagram below. What is the area, in cm^2, of the shaded region?



5. Two opposite corner dots from a 4 × 4 array have been removed, as shown in the diagram below. How many different squares can be formed using four of these 14 dots as vertices?



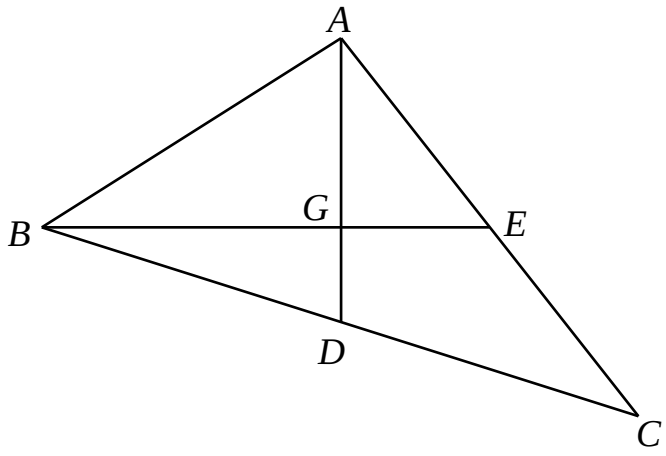
6. How many positive integers under 1000 with units digit 9 can be expressed as the sum of a power of 2 and a power of 3? Note that 1 is both a power of 2 and a power of 3.

7. Alice replaces each of the 2008 numbers 6, 7, 8, …, 2012, 2013 with the sum of its digits. Brian replaces each of Alice's numbers with the sum of its digits, and Colin replaces each of Brian's numbers with the sum of its digits. What is the number which Colin obtains most frequently?

8. What is the smallest positive integer which is 2 times the square of some positive integer and also 5 times the fifth power of some other positive integer?

9. Every positive integer can be expressed as a sum of distinct powers of 2. Note that 1 and 2 are powers of 2. How many three-digit numbers are sums of exactly 9 distinct powers of 2?

10. In triangle ABC, D is the midpoint of BC and E is the midpoint of CA. AD and BE are perpendicular to each other. The diagram below shows the point G where they intersect. This point is called the centroid of ABC, and has the property that $AG = 2DG$ and $BG = 2EG$. What is the value of $\dfrac{BC^2 + AC^2}{AB^2}$?

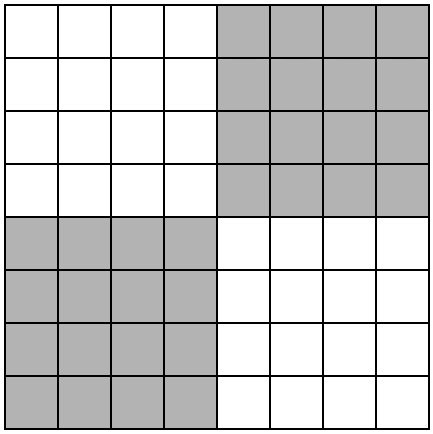


11. *O* is a point inside a quadrilateral *ABCD* such that its distances from the four vertices are 1, 2, 4 and 7 cm in some order. What is the maximum area, in cm^2, of *ABCD*?

12. From the product $1 \times 2 \times \cdots \times 2013$, what is thesmallest number of factors we must remove in order for the units-digit of the product of the remaining factors to be 9?

13. A positive integer is said to be *strange* if in its prime factorization, all powers are odd. For instance, 22, 23 and 24 form a block of three consecutive strange numbers because $22 = 2^1 \times 11^1$, $23 = 23^1$ and $24 = 2^3 \times 3^1$. What is the greatest length of a block of consecutive strange numbers?

14. Half of the squares in an 8×8 board are shaded, as shown in the diagram below. What is the total number of 2×2, 4×4 and 6×6 subboards such that half of the squares in each are shaded?



15. A positive integer with at most 9 digits is said to be *good* if its units digit is 0 or 1, its tens digit is 0, 1 or 2, its hundreds digit is 0, 1, 2 or 3, its thousands digit is 0, 1, 2, 3, or 4, and so on. Thus the first ten good numbers are 1, 10, 11, 20, 21, 100, 101, 110, 111 and 120. What is the 100-th good number?