國際數學競賽
INTERNATIONAL MATHEMATICS COMPETITION
Changchun China, 27 July ~ 1 August 2015

Elementary Mathematics International Contest

Individual Contest

Time limit: 90 minutes

Instructions:

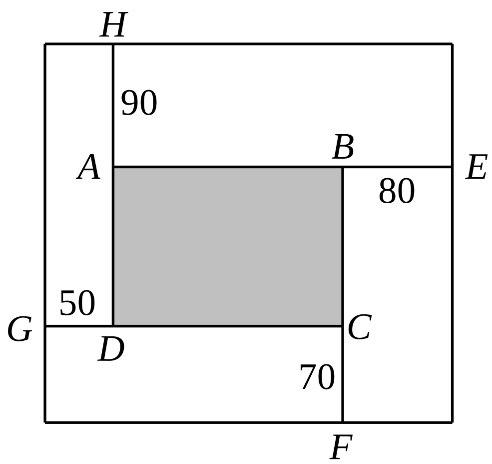
- Do not turn to the first page until you are told to do so.
- Write down your name, your contestant number and your team's name on the answer sheet.
- Write down all answers on the answer sheet. Only <u>Arabic NUMERICAL</u> answers are needed.
- Answer all 15 problems. Each problem is worth 10 points and the total is 150 points. For problems involving more than one answer, full credit will be given only if ALL answers are correct, no partial credit will be given. There is no penalty for a wrong answer.
- Diagrams shown may not be drawn to scale.
- No calculator, calculating device or protractor is allowed.
- Answer the problems with pencil, blue or black ball pen.
- All papers shall be collected at the end of this test.

English Version

1. Find the smallest four-digit number which has the same number of positive divisors as 2015.

2. Each students is asked to remove three of the first twenty-one positive integers (1, 2, 3, …, 21) and calculate the sum of the remaining eighteen numbers. The three numbers removed by each student contain two consecutive numbers, and no two students remove the same three numbers. At most how many students can correctly get 212 as the answer?

3. *ABCD* is a rectangular house. A fence extends *AB* to *E* with $BE = 80$ m. A fence extends *BC* to *F* with $CF = 70$ m. A fence extends *CD* to *G* with $DG = 50$ m. A fence extends *DA* to *H* with $AH = 90$ m. Fences through *E* and *G* parallel to *AD* and fences through *F* and *H* parallel to *AB* are built, enclosing a rectangular plot with four rectangular gardens around the house. The sum of the perimeters of the four gardens is 2016 m. What is the perimeter, in m, of the house?

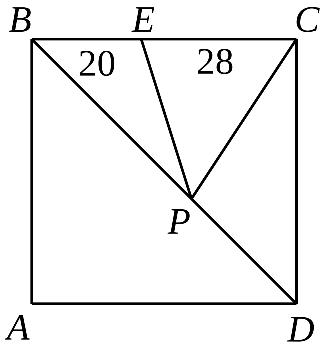


4. A certain community is divided into organizations, each organization is divided into associations, each association is divided into societies and each society is divided into clubs. The number of clubs in each society, the number of societies in each association and the number of associations in each organization are the same integer which is greater than 1. The community has a president, as does each organization, association, society and club. If there are 161 presidential positions altogether, how many organizations are there in this community?

5. Each of the machines A and B can produce one bottle per minute. Machine A has to rest for one minute after producing 3 bottles and Machine B has to rest 1.5 minutes after producing 5 bottles. What is the minimum number of minutes for these two machines to produce 2015 bottles together?

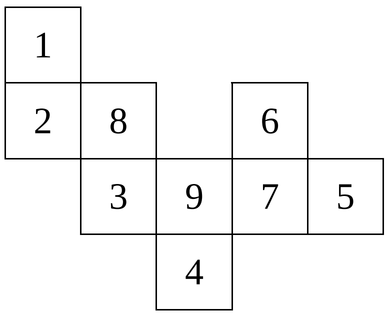
6. The six digits 1, 2, 3, 4, 5 and 6 are used to construct a one-digit number, a two-digit number and a three-digit number. Each must be used only once and all six digits must be used. The sum of the one-digit number and the two-digit number is 47 and the sum of the two-digit number and the three-digit number is 358. Find the sum of all the three numbers.

7. E is a point on the side BC of a square $ABCD$ such that $BE = 20$ cm and $CE = 28$ cm. P is a point of the diagonal BD. What is the smallest possible value, in cm, of $PE + PC$?



8. In a group of distinct positive integers, the largest one is less than 36, and is equal to three times the smallest one. The smallest number is equal to two-thirds of the group average. At most how many numbers are there in this group?

9. The diagram below shows the top view of a structure built with nine stacks of unit cubes. The number of cubes in each stack is indicated. Each stack rises from the bottom without gaps. The outside surface, including the nine 1 by 1 squares on the bottom, are painted. What is the total number of 1 by 1 faces that are painted?

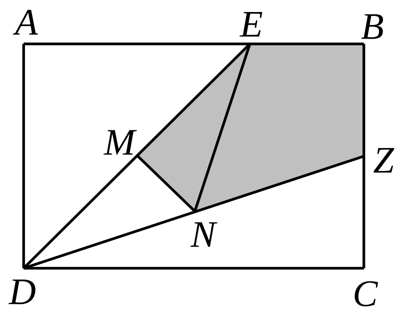


10. The sum of the digits of each of four different three-digit numbers is the same, and the sum of these four numbers is 2015. Find the sum of all possible values of the common digit sum of the four numbers.

11. There are three positive integers. The first is a two-digit number which consists of two identical digits. The second one is a two-digit number which consists of two different digits, and its units digit is the same as that of the first number. The third one is a one-digit number which consists of only one digit, which is the same as the tens digit of the second number. Exactly two of these three numbers are prime numbers. In how many different ways can the three positive integers be chosen?

12. A positive integer is divided by 5. The quotient and the remainder are recorded. The same number is divided by 3. Again the quotient and the remainder are recorded. If the same two numbers in different order are recorded, find the product of all possible values of the original number.

13. E is a point of the side AB of a rectangle $ABCD$ such that $AE = 2EB$, and Z is the midpoint of the side BC. M and N are the midpoints of DE and DZ respectively. If the area of the triangle EMN is 5 cm^2, calculate the area, in cm^2, of the pentagon $MEBZN$.



14. In how many ways can we divide the numbers 1, 2, 3, … , 12 into four groups, each containing three numbers whose sum is divisible by 3?

15. A number from 1, 2, 3, …, 19 is said to be a follower of a second number from 1, 2, 3, …, 19 if either the second number is 10 to 18 more than the first, or the first number is 1 to 9 more than the second. Thus 6 is a follower of 16, 17, 18, 19, 1, 2, 3, 4 and 5. In how many ways can we choose three numbers from 1, 2, 3, …, 19 such that the first is a follower of the second, the second is a follower of the third, and the **first** is also a follower of the **third**?