InIMC - 2017
India International Mathematics Competition
Organized by - CITY MONTESSORI INTER COLLEGE, RDSO Campus, Lucknow - INDIA
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Invitational World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes

Information:

- You are allowed 120 minutes for this paper, consisting of 12 questions in Section A to which only numerical answers are required, and 3 questions in Section B to which full solutions are required.
- Each question in Section A is worth 5 points. No partial credits are given. There are no penalties for incorrect answers, but you must not give more than the number of answers being asked for. For questions asking for several answers, full credit will only be given if all correct answers are found. Each question in Section B is worth 20 points. Partial credits may be awarded.
- Diagrams shown may not be drawn to scale.

Instructions:

- Write down your name, your contestant number and your team's name in the space provided on the first page of the question paper.
- For Section A, enter your answers in the space provided after the individual questions on the question paper. For Section B, write down your solutions on spaces provided after individual questions.
- You must use either a pencil or a ball-point pen which is either black or blue.
- You may not use instruments such as protractors, calculators and electronic devices.
- At the end of the contest, you must hand in the envelope containing the question paper and all scratch papers.

English Version

Team:______________ ***Name***:__________________ ***No.***: _____________ ***Score:***_________

For Juries Use Only

No.	Section A												Section B			Total	Sign by Jury
	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3		
Score																	
Score																	

Section A.

In this section, there are 12 questions, each correct answer is worth 5 points. Fill in your answer in the space provided at the end of each question.

1. An equal number of novels and textbooks are in hard covers; $\frac{2}{5}$ of the novels and $\frac{3}{4}$ of the textbooks are in hard covers. What fraction of the total number of books is in hard cover?

Answer : ____________

2. A farmer picks 2017 apples with an average weight of 100 grams. The average weight of all the apples heavier than 100 grams is 122 grams while the average weight of all the apples lighter than 100 grams is 77 grams. At least how many apples weighing exactly 100 grams did the farmer pick?

Answer : ________ apples

3. The sum of three sides of a rectangle is 2017 cm while the sum of the fourth side and the diagonal is also 2017 cm. Find the length, in cm, of the diagonal of the rectangle.

Answer : __________ cm

4. Let *a*, *b*, *c*, *d* be real numbers such that $0 \le a \le b \le c \le d$ and $c + d = a^2 + b^2 + c^2 + d^2 = 1$. Find the maximum value of $a + b$.

Answer : ____________

5. Find the least possible value of the fraction $\dfrac{a^2 + b^2 + c^2}{ab + bc}$ where *a*, *b* and *c* are positive real numbers.

Answer : ____________

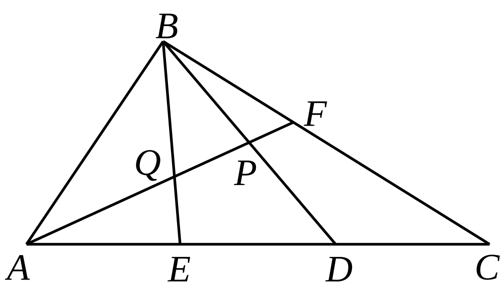
6. An octagon which has side lengths 3, 3, 11, 11, 15, 15, 15 and 15 cm is inscribed in a circle. What is the area, in cm^2, of the octagon?

Answer : __________ cm^2

7. If *x* and *y* are real numbers such that $4x^2 + y^2 = 4x - 2y + 7$, find the maximum value of $5x + 6y$.

Answer : ____________

8. In triangle *ABC*, points *E* and *D* are on side *AC* and point *F* is on side *BC* such that $AE = ED = DC$ and *BF* : *FC* = 2 : 3. *AF* intersects *BD* and *BE* at points *P* and *Q*, respectively. Find the ratio of the area *EDPQ* to the area of *ABC*.

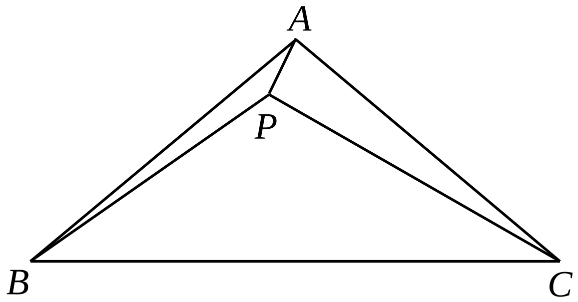


Answer : ____________

9. The sum of the non-negative real numbers x_1, x_2, …, x_8 is 8. Find the largest possible value of the expression $x_1x_2 + x_2x_3 + x_3x_4 + \cdots + x_7x_8$.

Answer : ____________

10. Let ABC be an isosceles triangle with $AB = AC$ and $\angle BAC = 100°$. A point P inside the triangle ABC satisfies that $\angle CBP = 35°$ and $\angle PCB = 30°$. Find the measure, in degrees, of angle $\angle BAP$.



Answer : ____________ °

11. If $xyz = -1$ and $a = x + \frac{1}{x}$, $b = y + \frac{1}{y}$, $c = z + \frac{1}{z}$, calculate $\sqrt{a^2 + b^2 + c^2 + abc}$.

Answer : ____________

12. Mal, Num, and Pin each have distinct number of marbles. Five times the sum of the product of the number of marbles of any two of them equals to seven times the product of the number of the marbles the three of them have. Find the largest possible sum of their marbles.

Answer : ____________

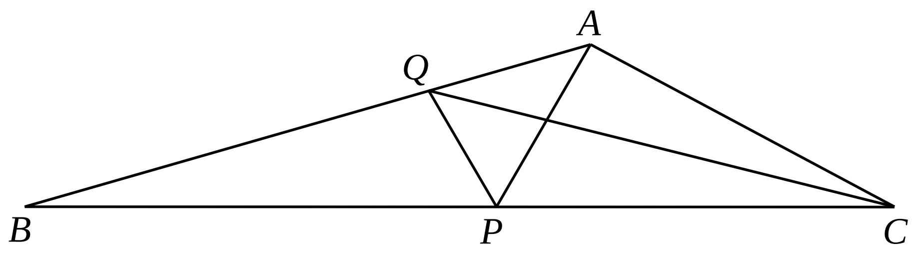
Section B.

Answer the following 3 questions, each question is worth 20 points. Show your detailed solution in the space provided.

1. Let x and y be non-negative integers such that $2^6 + 2^x + 2^{3y}$ is a perfect square and the expression should be less than 10,000. Find the maximum value of $x + y$.

Answer : ____________

2. Let ABC be a triangle such that $\angle B = 16°$ and $\angle C = 28°$. Let P be a point on BC such that $\angle BAP = 44°$ and let Q be a point on AB such that $\angle QCB = 14°$. Find, in degrees, $\angle PQC$.



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Answer : ______________

3. Let $f(x)$ and $g(x)$ be distinct quadratic polynomials such that the leading coefficients of both polynomials are equal to 1 and

$$f(1) + f(2017) + f(2017^2) = g(1) + g(2017) + g(2017^2).$$

Find x if $f(x) = g(x)$.

Answer : ______________