Invitational World Youth Mathematics Intercity Competition 2000

Individual Contest

Section A.
In this section, there are 12 questions. Fill in the correct answer in the space provided at the end of each question. Each correct answer is worth 5 points.

1. Find the unit digit of 17^{2000}.

2. The sum of four of the six fractions $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{9}$, $\frac{1}{12}$, $\frac{1}{15}$ and $\frac{1}{18}$ is equal to $\frac{2}{3}$. Find the product of the other two fractions.

3. Find the smallest odd three-digit multiple of 11 whose hundreds digit is greater than its units digit.

4. Find the sum of all the integers between 150 and 650 such that when each is divided by 10, the remainder is 4.

5. Find the quotient when a four-thousand-digit number consisting of two thousand 1s followed by two thousand 2s is divided by a two-thousand-digit numbers every digit of which is 6.

6. Find two unequal prime numbers p and q such that $p+q=192$ and $2p-q$ is as large as possible.

7. D is a point on the side BC of a triangle ABC such that $AC=CD$ and $\angle CAB = \angle ABC + 45°$. Find $\angle BAD$.

8. Let a, b, c, d and e be single-digit numbers. If the square of the fifteen-digit number $100000035811ab1$ is the twenty-nine-digit number $1000000cde22474824442657353 61$, find the value of $a+b+c-d-e$.

9. P is a point inside a rectangle $ABCD$. If $PA=4$, $PB=6$ and $PD=9$, find PC.

10. In the Celsius scale, water freezes at $0°$ and boils at $100°$. In the Sulesic scale, water freezes at $20°$ and boils at $160°$. Find the temperature in the Sulesic scale when it is $215°$ in the Celsius scale.

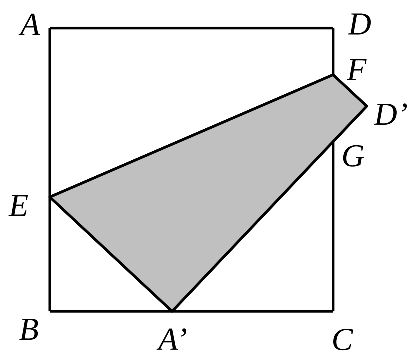
11. The vertices of a square all lie on a circle. Two adjacent vertices of another square lie on the same circle while the other two lie on one of its diameters. Find the ratio of the area of the second square to the area of the first square.

12. Ten positive integers are written in a row. The sum of any three adjacent numbers is 20. The first number is 2 and the ninth number is 8. Find the fifth number.

Section B.

Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. E is a point on the side AB and F is a point on the side CD of a square $ABCD$ such that when the square is folded along EF, the new position A' of A lies on BC. Let D' denote the new position of D and let G be the point of intersection of CF and $A'D'$. Prove that $A'E+FG=A'G$.



2. Twenty distinct positive integers are written on the front and back of ten cards, one on each face of every card. The sum of the two integers on each card is the same for all ten cards, and the sum of the ten integers on the front of the cards is equal to the sum of the ten integers on the back of the cards. The integers on the front of nine of the cards are 2, 5, 17, 21, 24, 31, 35, 36 and 42. Find the integer on the front of the remaining card.

3. Given are two three-digit numbers a and b and a four-digit number c. If the sums of the digits of the numbers $a+b$, $b+c$ and $c+a$ are all equal to 3, find the largest possible sum of the digits of the number $a+b+c$.