Invitational World Youth Mathematics Intercity Competition 2000

Team Contest

1. E is the midpoint of side BC of a square $ABCD$. H is the point on AE such that $BE = EH$. X is the point on AB such that $AH = AX$. Prove that : $AB \times BX = AX^2$.

2. Four non-negative integers have been entered in the following 5×5 table. Fill in the remaining 21 spaces with positive integers so that the sum of all the numbers in each row and in each column is the same.

	82			
				79
		103		
0				

3. For $n \geq 1$, define $a_n = 1000 + n^2$. Find the greatest value of the greatest common divisor of a_n and a_{n+1}.

4. Five teachers predict the order of finish of five classes A, B, C, D and E in an examination.

Guesses	**First**	**Second**	**Third**	**Fourth**	**Fifth**
Teacher 1	A	B	C	D	E
Teacher 2	E	D	A	B	C
Teacher 3	E	B	C	D	A
Teacher 4	C	E	D	A	B
Teacher 5	E	B	C	A	D

After the examination, which produces no ties between classes, it turns out that each of two teachers guesses correctly the ranks of two of the classes but is wrong about the ranks of the other three. The other three teachers are wrong about the rank of every class. Find the order of finish of the classes.

5. Find all triples (a, b, c) of positive integers such that $a \leq b \leq c$ and $\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)\left(1+\frac{1}{c}\right) = 2$.

6. Each team is given 50 square cardboard pieces and 50 equilateral triangular cardboard pieces. Using as many of these pieces as faces, construct a set of different convex polyhedra. Two polyhedra with the same numbers of vertices, edges, square faces and triangular faces are not considered different.