5^{th} Invitational World Youth Mathematics Inter-City Competition

第五屆青少年數學國際城市邀請賽

Team Contest 4^{th} August, 2004, Macau

Team: ________________________ Score: _________________

1. In right-angled triangle ΔABC, $\angle A = 30^\circ$, $BC = 1$, $\angle C = 90^\circ$. Consider all the equilateral triangles with all the vertices on the sides of the triangle ΔABC (i.e., the inscribed equilateral triangle of ΔABC). Determine the maximum area among all these equilateral triangles? Justify your answer.

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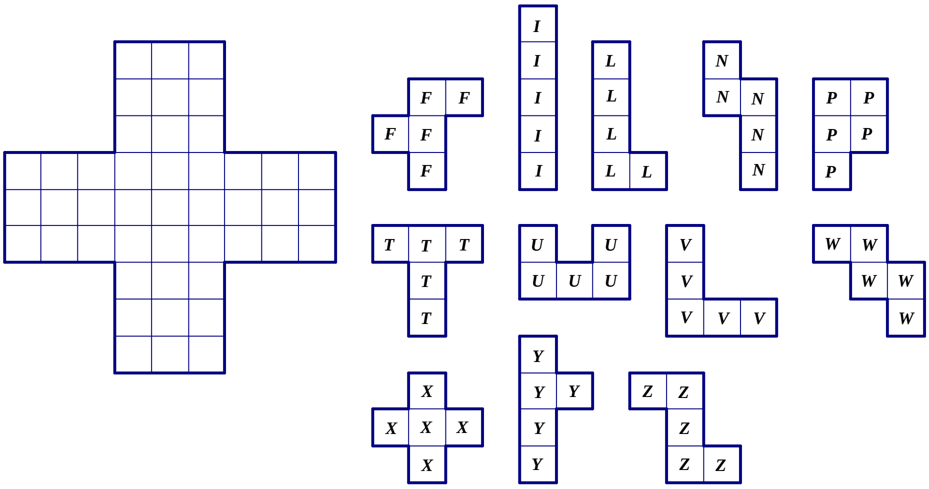
2. Below are the 12 pieces of pentominoes and a game board. Select four different pentominoes and place on the board so that all the other eight pieces can’t placed in this game board. The Pentominoes may be rotated and/or reflected and must follow the grid lines and no overlapping is allowed.



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3. Locate five buildings with heights 1, 2, 3, 4, 5 into every row and every column of the grid (figure A), once each. The numbers on the four sides in figure A below are the number of buildings that one can see from that side, looking row by row or column by column. One can see a building only when all the buildings in front of it are shorter. An example is given as shown in the figure B below, in which the number 5 is replaced by 4, under the similar conditions.

Answer:

			10			
	1				2	
12			5			11
	4				3	
			13			

Figure A

				8			
		3	2	1	2		
	3	1	3	4	2	2	
9	2	3	4	2	1	3	8
	3	2	1	3	4	1	
	1	4	2	1	3	2	
		1	2	3	2		
				8			

Figure B

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4. Let $|x|$ be the absolute value of real number x. Determine the minimum value of the expression $|25^n - 7^m - 3^m|$ where m and n can be any positive integers.

Team: ________________________ Score: _________________

5. There are m elevators in a building. Each of them will stop exactly in n floors and these floors does not necessarily to be consecutively. Not all the elevators start from the first floor. For any two floors, there is at least one elevator will stop on both floors. If m=11, n=3, determine the maximum number of floors in this building, and list out all the floors stop by each of these m elevators.

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6. In a soccer tournament, every team plays with other team once. In each game under the old scoring system, a winning team gains two points, and in the new score system, this team gains three points instead, while the losing team still get no points as before. A draw is worth one point for both teams without any changes. Is it possible for a team to be the winner of the tournament under the new system, and yet it finishes as the last placer under the old system? If this is possible, at least how many teams participate in this tournament, and list out the results of each game among those teams?

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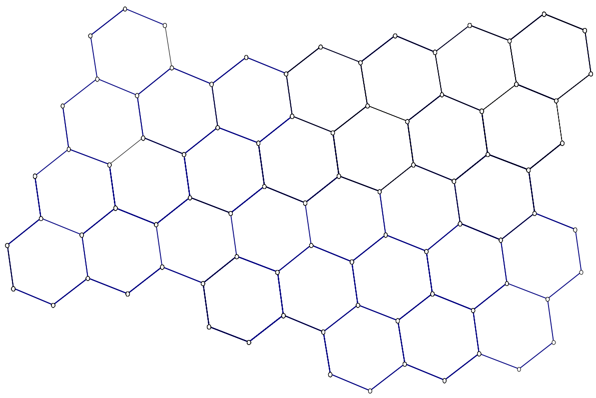
7. Determine the smallest integer *n* satisfying the following condition: one can divide the following figure into *n* ($n \geq 2$) congruent regions along the grid lines.

Answer:



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8. A polyomino is a figure formed of several unit squares joined along complete edges. Now one can only construct rectangle with at most 10 pieces of polyominoes where overlapping or gaps are not allowed, and satisfying the following conditions:
   a. the linear dimension of each piece, with at least one square, must be an integral multiple of the smallest piece, under rotation or reflection (if necessary);
   b. each piece is not rectangular;
   c. there are at least two pieces of different sizes.

The diagram on the left is a 9×4 rectangle constructed with six pieces of polyominoes while the diagram on the right is a 13×6 rectangle constructed with four pieces of polyominoes, but it does not satisfy the condition (a) stated above (namely the scale is not integral multiple).

Construct 10 rectangles with no two of them are similar and follow the rules stated above.