PEMIC PROBLEMS – Individual Contest

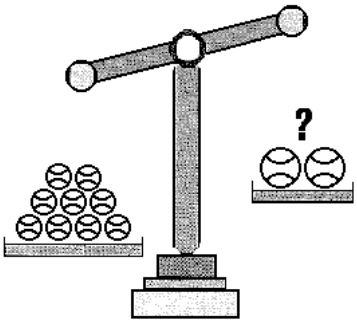
1. The numbers 4, 7, 10, 13, 16, …, where each number is three greater than the number preceding it, are written in order in a book, one hundred to a page. The first group of one hundred numbers begins on page 526. On which page will the number 2005 be located?

2. The numbers a, b, c, d, e, f and g are consecutive non-zero whole numbers arranged in increasing order. If $a + b + c + d + e + f + g$ is a perfect cube and $c + d + e$ is a perfect square, find the smallest possible value of d?

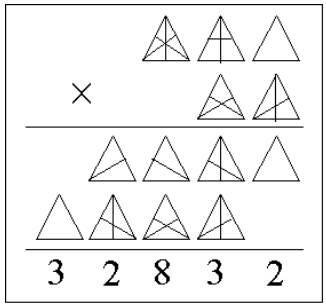
   (An example of a perfect cube is 8 because $8 = 2^3$.)

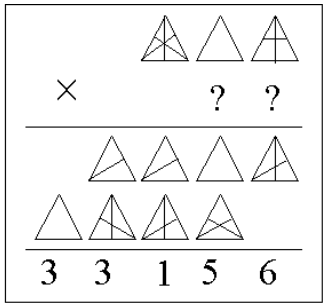
   (An example of a perfect square is 9 because $9 = 3^2$.)

3. If each large ball weighs $1\frac{1}{3}$ times the weight of each little ball, what is the minimum number of balls that need to be added to the right-hand side to make the scale balance? You may not remove balls, but only add small and/or large balls to the right-hand side.



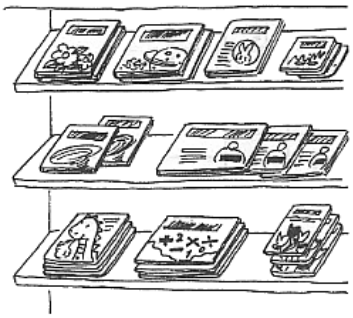
4. The different triangular symbols represent different digits from 1 to 9. The symbols represent the same digits in both examples. Find the two-digit number represented by **??.**





5. The following table shows the number of mathematics books sold over a period of five days. Find the number of books sold on Tuesday.

MATHEMATICS BOOKS

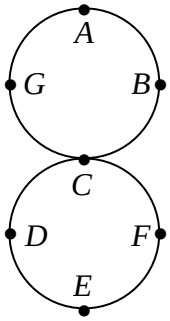


Monday, Tuesday & Wednesday	115
Wednesday & Thursday	85
Tuesday & Thursday	90
Monday & Friday	70
Thursday & Friday	80

6. Fractions in the form $\frac{a}{b}$ are created such that a and b are positive whole numbers and $a + b = 333$. How many such fractions are less than one and cannot be simplified?
(Cannot be simplified means that the numerator and denominator have no common factor)

7. Four friends were racing side by side down a dusty staircase. Peter went down two steps at a time, Bruce three steps at a time, Jessica four steps at a time, and Maitreyi five steps at a time. If the only steps with all four footprints were at the top and the bottom, how many steps had only one person's footprint?

8. In the diagram, there are two touching circles, each of radius 2 cm.
An ant starts at point *A* and walks around the figure 8 path ABCDEFCGA in that order. The ant repeats the figure 8 walk, again and again. After the ant has walked a distance of 2005π cm it becomes tired and stops. The ant stops at a point in the path. What letter point is it?



9. A basket and 16 potatoes are placed in a straight line at equal intervals of 6 meters, with the basket fixed at one end. What is the shortest possible time for Jose to bring the potatoes one by one into the basket, if he starts from where the basket is and runs at an average speed of 3 meters per second?

10. A sequence of digits is formed by writing the digits from the natural numbers in the order that they appear. The sequence starts:

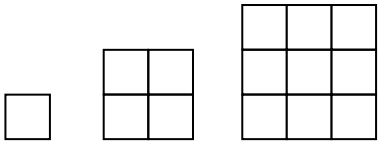
123456789101112 …

What is the 2005^{th} digit in the sequence?

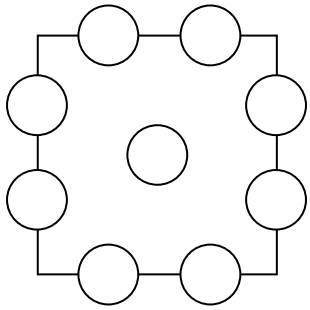
11. While *B* is riding a bicycle from Point *X* to Point *Y*, *C* is driving a car from Point *Y* to Point *X*, each at a steady speed along the same road. They start at the same time and, after passing each other, *B* takes 25 times longer to complete the journey as *C*. Find the ratio of the speed of the bicycle to the speed of the car.

12. Ten whole numbers (not necessarily all different) have the property that if all but one of them are added, the possible sums (depending on which one is omitted) are: 82, 83, 84, 85, 87, 89, 90, 91, 92. The 10^{th} sum is a repetition of one of these. What is the sum of the ten whole numbers?

13. A sequence of squares is made of identical square tiles. The edge of each square is one tile length longer than the edge of the previous square. The first three squares are shown. How many more tiles does the 2005^{th} square have than the 2004^{th}?



14. Lucky, Michael, Nelson and Obet were good friends. Obet had no money. Michael gave one-fifth of his money to Obet. Lucky gave one-fourth of his money to Obet. Finally, Nelson gave one-third of his money to Obet. Obet received the same amount of money from each of them. What fraction of the group's total money did Obet have at the end?



15. Each of the numbers from 1 to 9 is placed, one per circle, into the pattern shown. The sums along each of the four sides are equal. How many different numbers can be placed in the middle circle to satisfy these conditions?