2006 Wenzhou Invitational World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes 2006/7/12 Wenzhou, China

Team:________________ Name:________________ Score:______________

Section I:

In this section, there are 12 questions, fill in the correct answers in the spaces provided at the end of each question. Each correct answer is worth 5 points.

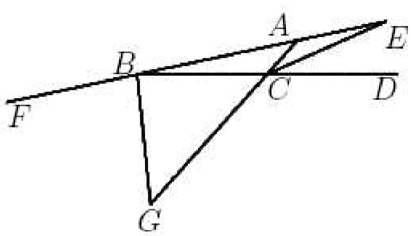
1. Colleen used a calculator to compute $\frac{a+b}{c}$, where *a*, *b* and *c* are positive integers. She pressed *a*, +, *b*, /, *c* and = in that order, and got the answer 11. When he pressed *b*, +, *a*, /, *c* and = in that order, she was surprised to get a different answer 14. Then she realized that the calculator performed the division before the addition. So she pressed (, *a*, +, *b*,), /, *c* and = in that order. She finally got the correct answer. What is it?

Answer:_______________

2. The segment *AB* has length 5. On a plane containing *AB*, how many straight lines are at a distance 2 from *A* and at a distance 3 from *B*?

Answer:_______________

3. In triangle *ABC*, *D* is a point on the extension of *BC*, and *F* is a point on the extension of *AB*. The bisector of $\angle ACD$ meets the extension of *BA* at *E*, and the bisector of $\angle FBC$ meets the extension of *AC* at *G*, as shown in the diagram below. If *CE* = *BC* = *BG*, what is the measure of $\angle ABC$?



Answer:_______________

4. The teacher said, "I have two numbers *a* and *b* which satisfy $a+b-ab=1$. I will tell you that *a* is not an integer. What can you say about *b*?" Alex said, "Then *b* is not an integer either." Brian said, "No, I think *b* must be some positive integer." Colin said, "No, I think *b* must be some negative integer." Who was right?

Answer:_______________

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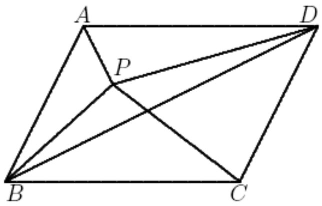
5. *ABCD* is a parallelogram and *P* is a point inside triangle *BAD*. If the area of triangle *PAB* is 2 and the area of triangle *PCB* is 5, what is the area of triangle *PBD*?



Answer:_______________

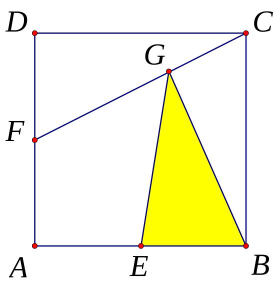
6. The non-zero numbers *a*, *b*, *c*, *d*, *x*, *y* and *z* are such that $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$. What is the value of $\frac{xyz(a+b)(b+c)(c+a)}{abc(x+y)(y+z)(z+x)}$?

Answer:_______________

7. On level ground, car travels at 63 kilometres per hour. Going uphill, it slows down to 56 kilometres per hour. Going downhill, it speeds up to 72 kilometres per hour. A trip from *A* to *B* by this car takes 4 hours, when the return trip from *B* to *A* takes 4 hours and 40 minutes. What is the distance between *A* and *B*?

Answer:_______________

8. The square *ABCD* has side length 2. *E* and *F* are the respective midpoints of *AB* and *AD*, and *G* is a point on *CF* such that 3 *CG* ＝2 *GF*. Determine the area of triangle *BEG*.



Answer:_______________

9. Determine *x*+*y* where *x* and *y* are real numbers such that $(2x+1)^2+y^2+(y-2x)^2=\frac{1}{3}$.

Answer:_______________

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10. A shredding company has many employees numbered 1, 2, 3, and so on along the disassembly line. The foreman receives a single-page document to be shredded. He rips it into 5 pieces and hands them to employee number 1. When employee n receives pieces of paper, he takes n of them and rips each piece into 5 pieces and passes all the pieces to employee n+1. What is the value of k such that employee k receives less than 2006 pieces of paper but hands over at least 2006 pieces?

Answer:_______________

11. A convex polyhedron Q is obtained from a convex polyhedron P with 36 edges as follows. For each vertex V of P, use a plane to slice off a pyramid with V as its vertex. These planes do not intersect inside P. Determine the number of edges of Q.

Answer:_______________

12. Let m and n be positive integers such that $\sqrt{m-174}+\sqrt{m+34}=n$. Determine the maximum value of n.

Answer:_______________

Section II:

Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. There are four elevators in a building. Each makes three stops, which do not have to be on consecutive floors or include the main floor. For any two floors, there is at least one elevator which stops on both of them. What is the maximum number of floors in this building?

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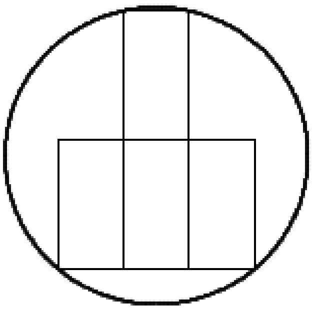
2. Four $2{\times}4$ rectangles are arranged as shown in the diagram below and may not be rearranged. What is the radius of the smallest circle which can cover all of them?



3. Partition the positive integers from 1 to 30 inclusive into k pairwise disjoint groups such that the sum of two distinct elements in a group is never the square of an integer. What is the minimum value of k?