2007 Changchun Invitational World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes 2007/7/23 Changchun, China

Team:_______________ Name:_______________ Score:_____________

Section I:

In this section, there are 12 questions, fill in the correct answers in the spaces provided at the end of each question. Each correct answer is worth 5 points.

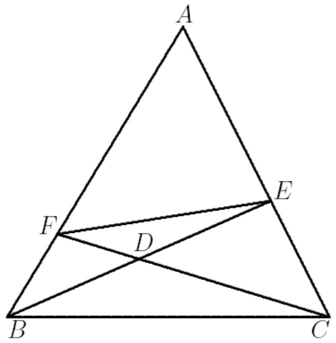
1. Let A_n be the average of the multiples of n between 1 and 101. Which is the largest among A_2, A_3, A_4, A_5 and A_6?

   Answer：_______________

2. It is a dark and stormy night. Four people must evacuate from an island to the mainland. The only link is a narrow bridge which allows passage of two people at a time. Moreover, the bridge must be illuminated, and the four people have only one lantern among them. After each passage to the mainland, if there are still people on the island, someone must bring the lantern back. Crossing the bridge individually, the four people take 2, 4, 8 and 16 minutes respectively. Crossing the bridge in pairs, the slower speed is used. What is the minimum time for the whole evacuation?

   Answer：_______________

3. In triangle *ABC*, *E* is a point on *AC* and *F* is a point on *AB*. *BE* and *CF* intersect at *D*. If the areas of triangles *BDF*, *BCD* and *CDE* are 3, 7 and 7 respectively, what is the area of the quadrilateral *AEDF*?



   Answer：_______________

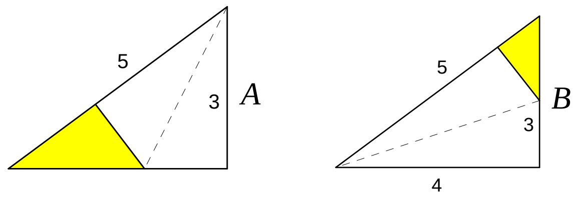
4. A regiment had 48 soldiers but only half of them had uniforms. During inspection, they form a 6×8 rectangle, and it was just enough to conceal in its interior everyone without a uniform. Later, some new soldiers joined the regiment, but again only half of them had uniforms. During the next inspection, they used a different rectangular formation, again just enough to conceal in its interior everyone without a uniform. How many new soldiers joined the regiment?

   Answer：_______________

5. The sum of 2008 consecutive positive integers is a perfect square. What is the minimum value of the largest of these integers?

Answer：________________

6. The diagram shows two identical triangular pieces of paper *A* and *B*. The side lengths of each triangle are 3, 4 and 5. Each triangle is folded along a line through a vertex, so that the two sides meeting at this vertex coincide. The regions not covered by the folded parts have respective areas S_A and S_B. If $S_A+S_B=39$, find the area of the original triangular piece of paper *A*.



Answer：________________

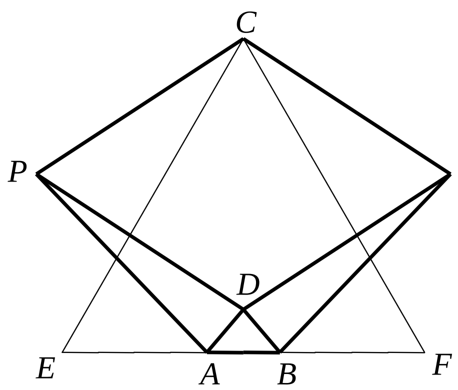
7. Find the largest positive integer *n* such that $3^{1024}-1$ is divisible by 2^n.

Answer：________________

8. A farmer use four straight fences, with respective lengths 1, 4, 7 and 8 units to form a quadrilateral. What is the maximum area of the quadrilateral the farmer can enclose?

Answer：________________

9. In the diagram, $CE=CF=EF$, $EA=BF=2AB$, and $PA=QB=PC=QC=PD=QD=1$, Determine *BD*.



Answer：________________

10. Each of the numbers 2, 3, 4, 5, 6, 7, 8 and 9 is used once to fill in one of the boxes in the equation below to make it correct. Of the three fractions being added, what is the value of the largest one?

$$\frac{1}{\square\times\square}+\frac{\square}{\square\times\square}+\frac{\square}{\square\times\square}=1$$

Answer：________________

11. Let x be a real number. Denote by $[x]$ the integer part of x and by $\{x\}$ the decimal part of x. Find the sum of all positive numbers satisfying $25\{x\}+[x]=125$.

Answer：________________

12. A positive integer n is said to be good if there exists a perfect square whose sum of digits in base 10 is equal to n. For instance, 13 is good because $7^2=49$ and $4+9=13$. How many good numbers are among 1, 2, 3, …, 2007?

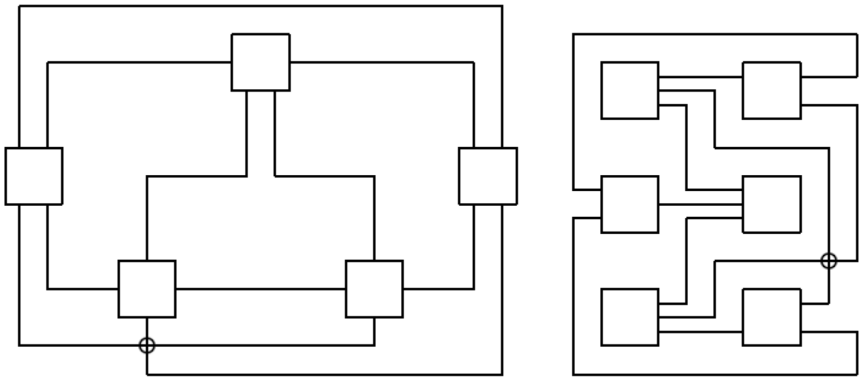
Answer：________________

Section II:

Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. A 4×4 table has 18 lines, consisting of the 4 rows, the 4 columns, 5 diagonals running from southwest to northeast, and 5 diagonals running from northwest to southeast. A diagonal may have 2, 3 or 4 squares. Ten counters are to be placed, one in each of ten of the sixteen cells. Each line which contains an even number of counters scores a point. What is the largest possible score?

2. There are ten roads linking all possible pairs of five cities. It is known that there is at least one crossing of two roads, as illustrated in the diagram below on the left. There are nine roads linking each of three cities to each of three towns. It is known that there is also at least one crossing of two roads, as illustrated in the diagram below on the right. Of the fifteen roads linking all possible pairs of six cities, what is the minimum number of crossings of two roads?



3. A prime number is called an ***absolute prime*** if every permutation of its digits in base 10 is also a prime number. For example: 2, 3, 5, 7, 11, 13 (31), 17 (71), 37 (73) 79 (97), 113 (131, 311), 199 (919, 991) and 337 (373, 733) are absolute primes. Prove that no ***absolute prime*** contains all of the digits 1, 3, 7 and 9 in base 10.