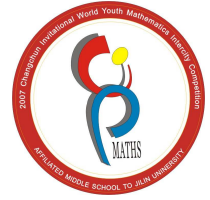# 2007 Changchun Invitational World Youth Mathematics Intercity Competition
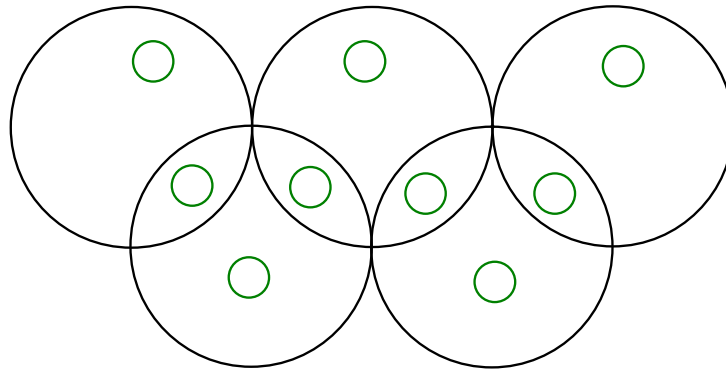
## Team Contest
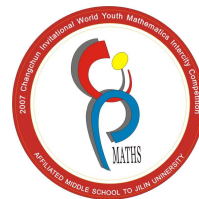
2007/7/23  Changchun, China

**Team**: _____     **Score**: _____

1.  Use each of the numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9 exactly once to fill in the nine small circles in the Olympic symbol below, so that the sum of all the numbers inside each large circle is 14. Write down the correct number in each small circle.
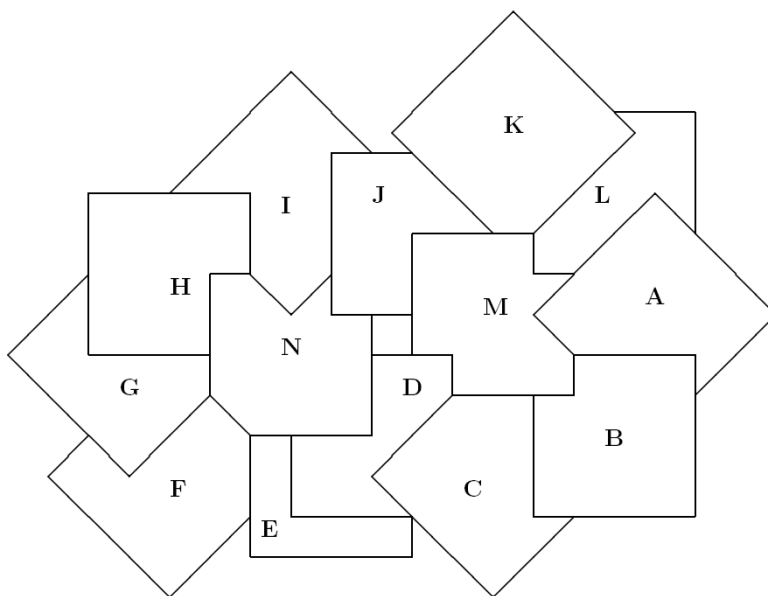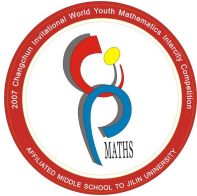
# Team Contest

2007/7/23   Changchun, China

**Team**: _____   **Score**: _____

2.   The diagram below shows fourteen pieces of paper stacked on top of one another. Beginning on the pieces marked B, move from piece to adjacent piece in order to finish at the piece marked F. The path must alternately climb up to a piece of paper stacked higher and come down to a piece of paper stacked lower. The same piece may be visited more than once, and it is not necessary to visit every piece. List the pieces of paper in the order visited.
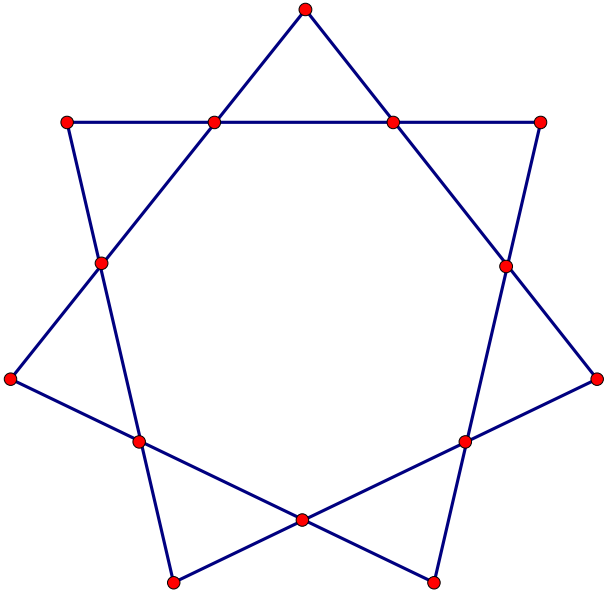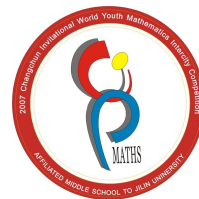
# Team Contest

2007/7/23 Changchun, China

**Team**: _____  **Score**: _____

3. There are 14 points of intersection in the seven-pointed star in the diagram on the below. Label these points with the numbers 1, 2, 3, …, 14 such that the sum of the labels of the four points on each line is the same. Give one set of solution, no explanation needed.

## Team Contest

2007/7/23   Changchun, China

**Team**: _____   **Score**: _____

4.  Mary found a 3-digit number that, when multiplied by itself, produced a number which ended in her 3-digit number. What is the sum of all the distinct 3-digit numbers which have this property?
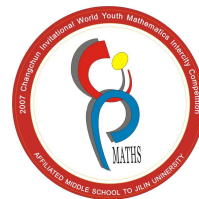
**Team Contest**                    2007/7/23   Changchun, China

**Team**: _____          **Score**: _____

5.   Determine all positive integers $m$ and $n$ such that $m^2+1$ is a prime number and $10(m^2+1)=n^2+1$.

## Team Contest

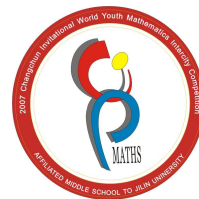2007/7/23   Changchun, China

**Team**: _____   **Score**: _____

6. Four teams take part in a week-long tournament in which every team plays every other team twice, and each team plays one game per day. The diagram below on the left shows the final scoreboard, part of which has broken off into four pieces, as shown on the diagram below on the right. These pieces are printed only on one side. A black circle indicates a victory and a white circle indicates a defeat. Which team wins the tournament?
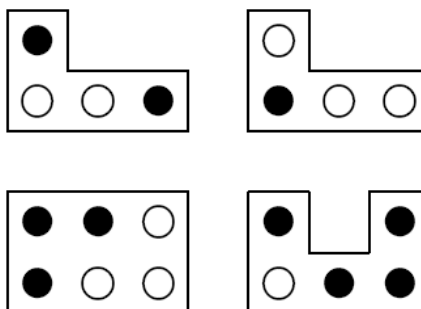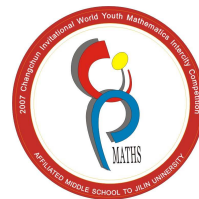
**Team Contest**                    2007/7/23   Changchun, China

**Team**: _____        **Score**: _____

7.  Let $A$ be a 3 by 3 array consisting of the numbers 1, 2, 3, …, 9. Compute the sum of the three numbers on the $i$-th row of $A$ and the sum of the three numbers on the $j$-th column of $A$. The number at the intersection of the $i$-th row and the $j$-th column of another 3 by 3 array $B$ is equal to the absolute difference of the two sums of array $A$.   For Example,

$$b_{12} = \left| \left( a_{11} + a_{12} + a_{13} \right) - \left( a_{12} + a_{22} + a_{32} \right) \right|.$$

Is it possible to arrange the numbers in array $A$ so that the numbers 1, 2, 3, …, 9 will also appear in array $B$?

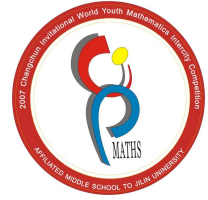| $a_{11}$ | $a_{12}$ | $a_{13}$ |
|---|---|---|
| $a_{21}$ | $a_{22}$ | $a_{23}$ |
| $a_{31}$ | $a_{32}$ | $a_{33}$ |

$A$

| $b_{11}$ | $b_{12}$ | $b_{13}$ |
|---|---|---|
| $b_{21}$ | $b_{22}$ | $b_{23}$ |
| $b_{31}$ | $b_{32}$ | $b_{33}$ |

$B$

**Team Contest**                                    2007/7/23   Changchun, China
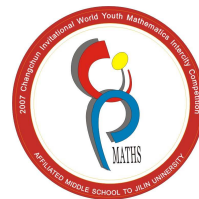
**Team**: _____        **Score**: _____

8.  The diagonals $AC$ and $BD$ of a convex quadrilateral are perpendicular to each other.  Draw a line that passes through point $M$, the midpoint of $AB$ and perpendicular to $CD$; draw another line through point $N$, the midpoint of $AD$ and perpendicular to $CB$. Prove that the point of intersection of these two lines lies on the line $AC$.

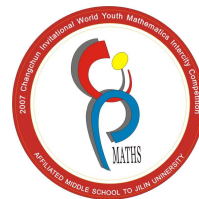**Team Contest**                    2007/7/23   Changchun, China

**Team**: _____          **Score**: _____

9.   The positive integers from 1 to $n$ (where $n>1$) are arranged in a line such that the sum of any two adjacent numbers is a square. What is the minimum value of $n$?

# 2007 Changchun Invitational World Youth Mathematics Intercity Competition

## Team Contest

2007/7/23   Changchun, China

**Team**: _____   **Score**: _____

10. Use one of the five colours (R represent red, Y represent yellow, B represent blue, G represent green and W represent white) to paint each square of an 8×8 chessboard, as shown in the diagram below. Then paint the rest of the squares so that all the squares of the same colour are connected to one another edge to edge. What is the largest number of squares of the same colour as compare to the other colours?

| R |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|
|   |   |   |   |   |   | Y |   |
|   |   | B |   |   |   |   |   |
| G |   |   |   |   |   |   | G |
|   |   |   | R |   |   |   |   |
|   | W |   |   |   |   | W |   |
|   |   |   |   |   |   |   |   |
|   |   | B | Y |   |   |   |   |