International Mathematics Competition,

25~29 July, 2010, Incheon, Korea,

Invitational World Youth Mathematics Intercity Competition

Team Contest

Instructions:

- Do not turn to the first page until you are told to do so.
- Remember to write down your team name in the space indicated on the first page.
- There are 10 problems in the Team Contest, arranged in increasing order of difficulty. Each question is printed on a separate sheet of paper. Each problem is worth 40 points and complete solutions of problem 1, 2, 3, 5, 6, 7 and 10 are required for full credits. Partial credits may be awarded. In case the spaces provided in each problem are not enough, you may continue you work at the back page of the paper. Only answers are required for Problem number 4, 8 and 9.The four team members are allowed 10 minutes to discuss and distribute the first 8 problems among themselves. Each student must solve at least one problem by themselves. Each will then have 35 minutes to write the solutions of their allotted problem independently with no further discussion or exchange of problems. The four team members are allowed 15 minutes to solve the last 2 problems together.
- No calculator or calculating device or electronic devices are allowed.
- Answer must be in pencil or in blue or black ball point pen.
- All papers shall be collected at the end of this test.

English Version

TEAM CONTEST

Team：____________________________ Score：_______________

1. Solve the following system of equations for real numbers *w*, *x*, *y* and *z*:

$$\begin{cases} w+8x+3y+5z=20 \\ 4w+7x+2y+3z=-20 \\ 6w+3x+8y+7z=20 \\ 7w+2x+7y+3z=-20. \end{cases}$$

ANSWER: $w=$ $x=$ $y=$ $z=$ ____________________________________

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2. In the convex quadrilateral *ABCD*, *AB* is the shortest side and *CD* is the longest. Prove that $\angle A > \angle C$ and $\angle B > \angle D$.

ANSWER: ______________________________________

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3. Let $m \geq n$ be integers such that $m^3 + n^3 + 1 = 4mn$. Determine the maximum value of $m - n$.

ANSWER: ______________________________________

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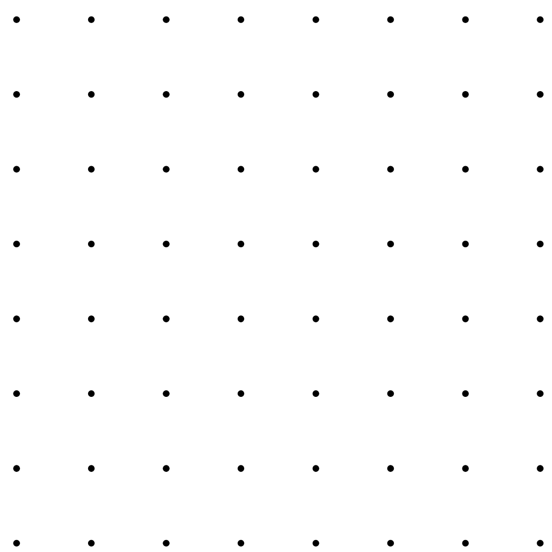
4. Arranged in an 8×8 array are 64 dots. The distance between adjacent dots on the same row or column is 1 cm. Determine the number of rectangles of area 12 cm^2 having all four vertices among these 64 dots.



ANSWER: ______________________________

TEAM CONTEST

Team：________________________ Score：______________

5. Determine the largest positive integer n such that there exists a unique positive integer k satisfying $\frac{8}{15}<\frac{n}{n+k}<\frac{7}{13}$.

ANSWER: ______________________________

TEAM CONTEST

Team：____________________________ Scroe：_______________

6. In a 9×9 table, every square contains a number. In each row and each column at most four different numbers appear. Determine the maximum number of different numbers that can appear in this table.

ANSWER: ____________________________________

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7. In a convex quadrilateral $ABCD$, $\angle ABD = 16°$, $\angle DBC = 48°$, $\angle BCA = 58°$ and $\angle ACD = 30°$. Determine $\angle ADB$, in degree.

ANSWER: ______________________________

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Team : ______________________ Score : ______________

8. Determine all ordered triples (x, y, z) of positive rational numbers such that each of $x+\frac{1}{y}$, $y+\frac{1}{z}$ and $z+\frac{1}{x}$ is an integer.

ANSWER: ______________________________

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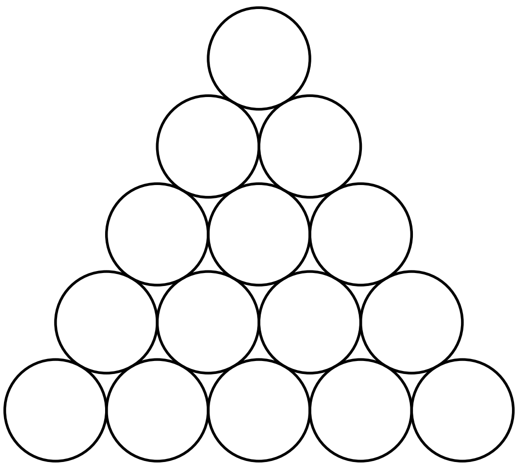
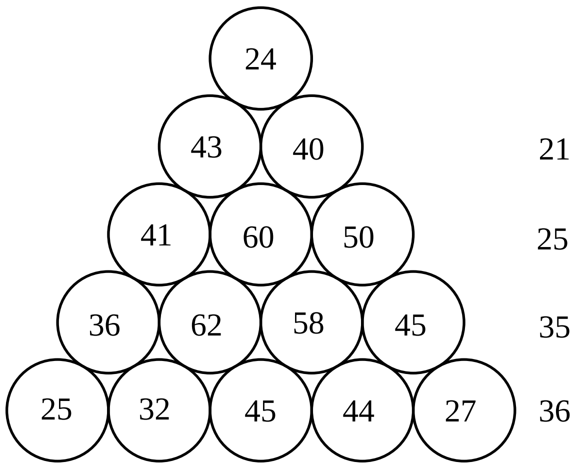
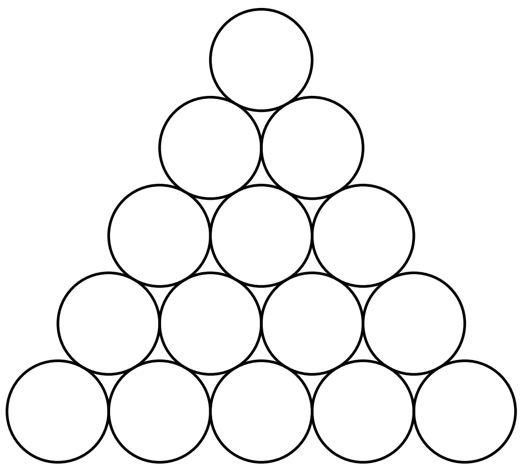
9. Assign each of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 and 15 into one of the fifteen different circles in the diagram shown below on the left, so that
   (a) the number which appear in each circle in the diagram below on the right represents the sum of the numbers which will be in that particular circle and all circles touching it in the diagram below on the left;
   (b) except the number in the first row, the sum of the numbers which will be in the circles in each row in the diagram below on the left is located at the rightmost column in the diagram below on the right.





ANSWER: ____________________________________



TEAM CONTEST

Team：____________________________ Score：_______________

10. The letters K, O, R, E, A, I, M and C are written in eight rows, with 1 K in the first row, 2 Os in the second row, and so on, up to 8 Cs in the last row. Starting with the lone K at the top, try to spell the "words" KOREA IMC by moving from row to row, going to the letter directly below or either of its neighbours, as illustrated by the path in boldface. It turns out that one of these 36 letters may not be used. As a result, the total number of ways of spelling KOREA IMC drops to 516. Circle the letter which may not be used.

K
O O
R **R** R
E **E** E E
A A **A** A A
I I I **I** I I
M M **M** M M M M
C C C **C** C C C C

ANSWER:

K
O O
R R R
E E E E
A A A A A
I I I I I I
M M M M M M M
C C C C C C C C