## Elementary Mathematics International Contest

## Individual Contest

## Time limit: 90 minutes

## Instructions:

- Do not turn to the first page until you are told to do so.
- Write down your name, your contestant number and your team's name on the answer sheet.
- Write down all answers on the answer sheet. Only Arabic NUMERICAL answers are needed.
- Answer all 15 problems. Each problem is worth 10 points and the total is 150 points. For problems involving more than one answer, full credit will be given only if ALL answers are correct, no partial credit will be given. There is no penalty for a wrong answer.
- Diagrams shown may not be drawn to scale.
- No calculator or calculating device is allowed.
- Answer the problems with pencil, blue or black ball pen.
- All papers shall be collected at the end of this test.


## English Version

## Elementary Mathematics International Contest Individual Contest

Time limit: 90 minutes $\quad 27^{\text {th }}$ July 2010 Incheon, Korea

1. A computer billboard is displaying the three "words" : IMC 2010 INCHEON. A malfunction causes the initial "letter" of each of the three words to be shifted to the end of that word every minute. Thus after 1 minute, the billboard reads MCI 0102 NCHEONI, and after 2 minutes, it reads CIM 1020 CHEONIN. After how many minutes will the original three words reappear for the first time?
2. What is the sum of the digits of the number $10^{2010}-2010$ ?
3. By the notation $d_{n}$, we mean an $n$-digit number consisting of $n$ times of the digit $d$. Thus $5_{3}=555$ and $4_{3} 9_{5} 8_{1} 3_{6}=444999998333333$. If $2_{w} 3_{x} 5_{y}+3_{y} 5_{w} 2_{x}=$ $5_{3} 7_{2} 8_{z} 5_{1} 7_{3}$ for some integers $w, x, y$ and $z$, what is the value of $w+x+y+z$ ?
4. A man weighs 60 kg plus one-quarter of his weight. His wife weighs 64 kg plus one-fifth of her weight. What is the absolute difference between the weights of the man and his wife in kg ?
5. In quadrilateral $A B C D, A B=6 \mathrm{~cm}, A D=4 \mathrm{~cm}, B C=7 \mathrm{~cm}$ and $C D=15 \mathrm{~cm}$. If the length of $A C$ is an integer number of cm , what is this number?

6. The speed of the current in the river is 1 km per hour. A man rows a boat at constant speed. He rows upstreams for 3 hours, and rows downstreams for 2 hours to return to his starting point. What is the distance, in km, between the starting point of the boat and the point at which the boat turns around?
7. In the quadrilateral $A B C D, A B$ is parallel to $D C$ and $A D=B C$. If eight copies of this quadrilateral can be used to form a hollow regular octagon as shown in the diagram below, what is the measure of $\angle B A D$, in degree ?

8. Let $\overline{a b c}, \overline{d e f}$ be two different 3-digit numbers. If the difference $\overline{a b c d e f}-\overline{\text { defabc }}$ is divisible by 2010, what is the largest possible sum of these two 3-digit numbers?
9. What is the average of all different 9-digits numbers where each consists of the digit 5 five times and the digit 4 four times?
10. $A B C D$ is a rectangle with $A B=4 \mathrm{~cm}$ and $B C=6 \mathrm{~cm} . E, F, G$ and $H$ are points on the sides $A B, B C, C D$ and $D A$ respectively, such that $A E=C G=3 \mathrm{~cm}$ and $B F=D H=4 \mathrm{~cm}$. If $P$ is a point inside $A B C D$ such that the area of the quadrilateral $A E P H$ is $5 \mathrm{~cm}^{2}$, what is the area the quadrilateral $P F C G$, in $\mathrm{cm}^{2}$ ?

11. Narrow vegetable spring-rolls of length 8 cm are supposed to be made by rolling $8-\mathrm{cm}$ bean sprouts inside $6 \mathrm{~cm} \times 8 \mathrm{~cm}$ rice papers into cylinders. Instead, the workers are provided with 6 cm bean sprouts. So they roll the rice paper the other way and get wide cylinders of length 6 cm . For either kind of spring rolls, there is an overlap of 1 cm in order for the rice paper to stick. What is the ratio of the volume of the 8 cm spring roll to the volume of the 6 cm spring roll?
12. The largest of 23 consecutive odd numbers is 5 times the smallest. What is the average of these 23 numbers?
13. The digits $1,2,3,4,5,6,7,8$, and 9 are to be written in the squares so that every row and every column with three numbers has a total of 13 . Two numbers have already been entered. What is the number in the square marked $*$ ?

14. In a test given in four subjects, each of five students obtained a score of $w, x, y$ or $z$ in each individual subject, as shown in the table below. The total score of each student had been computed, as well as the class total for each subject except for one. What was the class total for Biology?

| Students | Anna | Gail | Mary | Patty | Susie | Class Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Algebra | $w$ | $z$ | $w$ | $z$ | $y$ | 416 |
| Biology | $w$ | $x$ | $y$ | $y$ | $z$ | $?$ |
| Chemistry | $x$ | $y$ | $y$ | $w$ | $x$ | 428 |
| Dictation | $y$ | $w$ | $z$ | $z$ | $x$ | 401 |
| Individual Total | 349 | 330 | 349 | 326 | 315 |  |

15. What is the largest positive integer $n$ which does not contain the digit 0 , such that the sum of its digits is 15 and the sum of the digits of $2 n$ is less than 20 ?
