Ministry of National Education
Republic of Indonesia

Indonesia International Mathematics Competition 2011

Invitational World Youth Mathematics Intercity Competition

Individual Contest

Instructions:

- Do not turn to the first page until you are told to do so.
- Remember to write down your team name, your name and contestant number in the spaces indicated on the first page.
- The Individual Contest is composed of two sections with a total of 120 points.
- Section A consists of 12 questions in which blanks are to be filled in and only **ARABIC NUMERAL** answers are required. For problems involving more than one answer, points are given only when **ALL** answers are correct. Each question is worth 5 points. There is no penalty for a wrong answer.
- Section B consists of 3 problems of a computational nature, and the solutions should include detailed explanations. Each problem is worth 20 points, and partial credit may be awarded.
- You have a total of 120 minutes to complete the competition.
- No calculator, calculating device, watches or electronic devices are allowed.
- Answers must be in pencil or in blue or black ball point pen.
- All papers shall be collected at the end of this test.

English Version

Invitational World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes 20th July 2011 Bali, Indonesia

Team: ______________ Name: __________________ No.: _____________ Score: _________

Section A.

In this section, there are 12 questions. Fill in the correct answer in the space provided at the end of each question. Each correct answer is worth 5 points.

1. Let a, b, and c be positive integers such that

$$\begin{cases} ab+bc+ca+2(a+b+c)=8045, \\ abc-a-b-c=-2. \end{cases}$$

Find the value of $a+b+c$.

Answer : _________

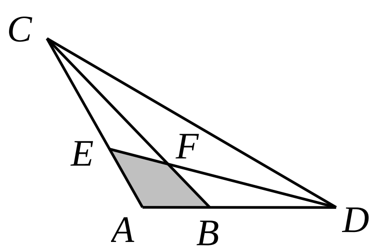
2. There are two kinds of students in a certain class, those who always lie and those who never lie. Each student knows what kind each of the other students is. In a meeting today, each student tells what kind each of the other students is. The answer “liar” is given 240 times. Yesterday a similar meeting took place, but one of the students did not attend. The answer “liar” was given 216 times then. How many students are present today?

Answer : __________

3. The product $1!\times2!\times\ldots\times2011!\times2012!$ is written on the blackboard. Which factor, in term of a factorial of an integer, should be erased so that the product of the remaining factors is the square of an integer? (The factorial sign $n!$ stands for the product of all positive integers less than or equal to n.)

Answer : __________

4. B and E are points on the sides AD and AC of triangles ACD such that BC and DE intersect at F. Triangles ABC and AED are congruent. Moreover, $AB=AE=1$ and $AC=AD=3$. Determine the ratio between the areas of the quadrilateral $ABFE$ and the triangle ADC.



Answer : __________

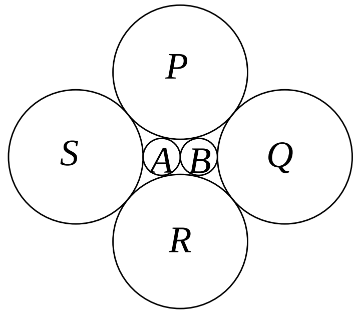
5. A positive integer n has exactly 4 positive divisors, including 1 and n. Furthermore, $n+1$ is four times the sum of the other two divisors. Find n.

Answer : __________

6. Jo tells Kate that the product of three positive integers is 36. Jo also tells her what the sum of the three numbers is, but Kate still does not know what the three numbers are. What is the sum of the three numbers?

Answer : ________

7. Two circles A and B, both with radius 1, touch each other externally. Four circles P, Q, R and S, all with the same radius r, are such that P touches A, B, Q and S externally; Q touches P, B and R externally; R touches A, B, Q and S externally; and S touches P, A and R externally. Calculate r.



Answer : ________

8. Find the smallest positive common multiple of 7 and 8 such that each digit is either 7 or 8, there is at least one 7 and there is at least one 8.

Answer : ________

9. The side lengths of a triangle are 50 cm, 120 cm and 130 cm. Find the area of the region consisting of all the points, inside and outside the triangle, whose distances from at least one point on the sides of the triangle are 2 cm. Take $\pi = \dfrac{22}{7}$.

Answer : ________

10. Find the number of positive integers which satisfy the following conditions:
    (1) It contains 8 digits each of which is 0 or 1.
    (2) The first digit is 1.
    (3) The sum of the digits on the even places equals the sum of the digits on the odd places.

Answer : ________

11. A checker is placed on a square of an infinite checkerboard, where each square is 1 cm by 1 cm. It moves according to the following rules:
- In the first move, the checker moves 1 square North.
- All odd numbered moves are North or South and all even numbered moves are East or West.
- In the n-th move, the checker makes a move of n squares in the same direction.

The checker makes 12 moves so that the distance between the centres of its initial and final squares is as small as possible. What is this minimum distance?

Answer : ________ cm

12. Let a, b and c be three real numbers such that

$$\frac{a(b-c)}{b(c-a)} = \frac{b(c-a)}{c(b-a)} = k > 0$$

for some constant k. Find the greatest integer less than or equal to k.

Answer : ________

Section B.

Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. The diagonals AC and BD of a quadrilateral $ABCD$ intersect at a point E. If $AE=CE$ and $\angle ABC=\angle ADC$, does $ABCD$ have to be a parallelogram?

2. When a=1, 2, 3, …, 2010, 2011, the roots of the equation $x^2-2x-a^2-a=0$ are $(\alpha_1,\beta_1),(\alpha_2,\beta_2),(\alpha_3,\beta_3),\cdots,(\alpha_{2010},\beta_{2010}),(\alpha_{2011},\beta_{2011})$ respectively.

Evaluate $\dfrac{1}{\alpha_1}+\dfrac{1}{\beta_1}+\dfrac{1}{\alpha_2}+\dfrac{1}{\beta_2}+\dfrac{1}{\alpha_3}+\dfrac{1}{\beta_3}+\cdots+\dfrac{1}{\alpha_{2010}}+\dfrac{1}{\beta_{2010}}+\dfrac{1}{\alpha_{2011}}+\dfrac{1}{\beta_{2011}}$.

3. Consider 15 rays that originate from one point. What is the maximum number of obtuse angles they can form? (The angle between any two rays is taken to be less than or equal to 180°)