Solution to First Round of 2013 IMAS Upper Primary Division

1. What is the value of the expression $3\times11\times61+3+11+61$?
   (A) 2013 (B) 2088 (C) 2113
   (D) 4026 (E) 4052169

【Suggested Solution #1】

Since $3\times11\times61+3+11+61=2013+3+11+61=2088$.

【Suggested Solution #2】

It is clear that $3\times11\times61+3+11+61$ is the sum of four odd numbers. So the answer will be an even number, then the options (A), (C) and (E) cannot be the answers, but $3\times11\times61+3+11+61\le40\times61+100\le2540$, it follows that option D cannot be the answer. Hence, we choose (B).

Answer: B

2. Which of the following is the closest length of time for one day?
   (A) 0.9 day (B) 1.2 day (C) 23 hours
   (D) 26 hours (E) 1410 minutes

【Suggested Solution】

Let us convert all the time measurements into minutes. 1 day = 1440 minutes, 0.9 day = 1296 minutes, 1.2 day = 1728 minutes, 23 hours = 1380 minutes, 26 hours = 1560 minutes and

$$1440-1296=144$$
$$1728-1440=288$$
$$1440-1380=60$$
$$1560-1440=120$$
$$1440-1410=30$$

Hence, 1410 minutes is closest to 1440 minutes, which is nearer to 1 day.

Answer: E

3. Walter has two options in going to school. He can (a) walk 3 minutes to the bus stop and ride the bus for 15 minutes to the school, or (b) walk 5 minutes to the LRT station, ride a train for 6 minutes and walk 5 minutes to school. If he does not have to wait for the bus at the bus stop, nor the train on the LRT station, what is the minimum number of minutes required for him to get to school?
   (A) 15 (B) 16 (C) 17 (D) 18 (E) 19

【Suggested Solution】

When Walter selects option (a) for going to school from his house, he needs a total 3+15=18 minutes. When he selects option (b), he needs 5+6+5=16 minutes. Hence, Walter needs at least 16 minutes in going to school.

Answer: B

4. Which of the following five numbers is divisible by 6?

(A) 98 (B) 163 (C) 192 (D) 212 (E) 254

【Suggested Solution】

A number is divisible by 6 if it is divisible both by 2 and 3. While a number is divisible by 2 if its last digit is an even number, hence option (B) is not the required number. Likewise, a number is divisible by 3 if the sum of the digits of the number is divisible by 3. After doing the divisibility test, 9+8=17, 1+9+2=12, 2+1+2=5, 2+5+4=11. Hence, only option (C) satisfies the Property of Divisibility.

Answer: C

5. Zachary has a computer program which accepts an input and produces an output. Some of the data are shown in the following table.

Input	1	2	3	4	5	6	7
Output	4	7	10	13	16	?	22

What is the output when the input is 6?

(A) 17 (B) 18 (C) 19 (D) 20 (E) 21

【Suggested Solution #1】

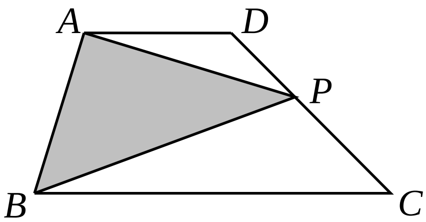
From the given information, it shows that the output data is 1 more than three times that of the input data. Thus, when we enter, the output will become $3 \times 6 + 1 = 19$.

【Suggested Solution #2】

From the given information, we discover that the output data is an arithmetic sequence which is composed of 4, 7, 10, 13, 16, ..., with a common difference of 3. Hence, when the input data is 6, the output data is $16 + 3 = 19$.

Answer: C

6. In the diagram, *AD* is parallel to *BC*. A point *P* moves from *C* to *D* along the side *CD*. Which of the following is the accurate description of the change in the area of triangle *ABP* during the motion?



(A) increasing (B) decreasing

(C) increasing then decreasing

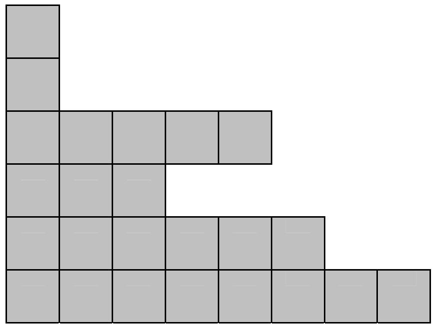
(D) decreasing then increasing (E) unchanged

【Suggested Solution】

Let us consider *AB* as the base of $\triangle ABP$, when point *P* is a moving point starting from point *C* to point *D*, the distance from point *P* to side *AB* keeps on decreasing, Hence, the area of $\triangle ABP$ becomes smaller and smaller.

Answer: B

7. Langford has a square piece of chocolate divided into unit squares. After eating some of the unit squares, what is left of the square piece is shown in the diagram. At least how many unit squares of chocolate Langford must have eaten?



（A）20 （B）24 （C）36

（D）40 （E）64

【Suggested Solution】

Let the side length of each small square chocolate as 1 unit. From the diagram, we can easily determine the side length of each side of the original big square chocolate is 8 units. Hence, we know there are 24 small un-eaten or remaining chocolates. So we conclude Langford has at least eaten $8 \times 8 - 24 = 40$ small squares of chocolate.

Answer: D

8. When $\frac{22}{7}$ is expressed as a decimal without rounding, how many decimal places should we take so that the positive difference of the result with 3.14159 is small as possible?

（A）1 （B）2 （C）3 （D）4 （E）5

【Suggested Solution】

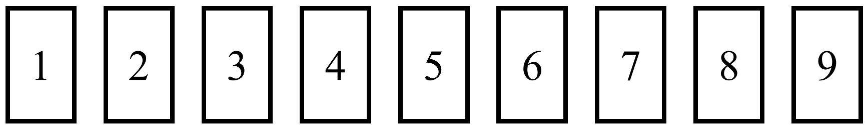
Since $\frac{22}{7} = 3.\overline{142857}$, so when we find the difference of 3.14159 with each of the rounding number of $\frac{22}{7}$,(that is; one digit at a time right after the decimal point of $\frac{22}{7}$), we have the following:

$$3.14159 - 3.1 = 0.04159$$
$$3.14159 - 3.14 = 0.00159$$
$$3.142 - 3.14159 = 0.00041$$
$$3.1428 - 3.14159 = 0.00121$$
$$3.14285 - 3.14159 = 0.00126$$
$$\vdots$$

From the above expressions, we know that the smallest positive difference between 3.14159 and each of the rounding off number one by one right after the decimal point of $\frac{22}{7}$ will happen when three digits is taken. The difference between 3.14159 and the other rounding off numbers of $\frac{22}{7}$ when 4 is taken and more digits will be increasing again. Hence, we must choose the option (C).

Answer: C

9. The diagram shows nine cards numbered 1 to 9. One of the cards is taken out, and the sum of the numbers of the remaining cards is 8 times that card. What is the number on the card that was taken out?



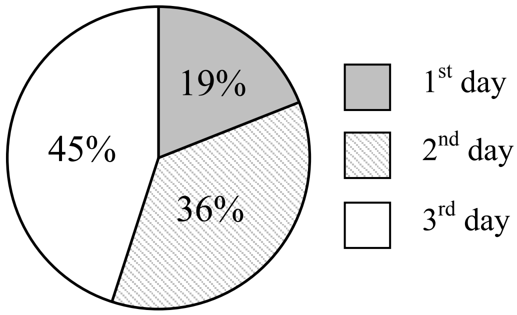
(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

【Suggested Solution】

From the given information, we know the sum of all numbers in those nine cards is 1 + 8 = 9 times the number on the card that will be pulled out. Since $1 + 2 + 3 + \cdots + 8 + 9 = 45 = 9 \times 5$, then the card that must have been taken out is the number 5.

Answer: A

10. Mrs. Huang spent three days making ornaments. The diagram shows the percentage of the number of ornaments she made on each day. If she made 152 ornaments on the first day, how many ornaments did she make on the third day?



(A) 190 (B) 360 (C) 450
(D) 720 (E) 800

【Suggested Solution #1】

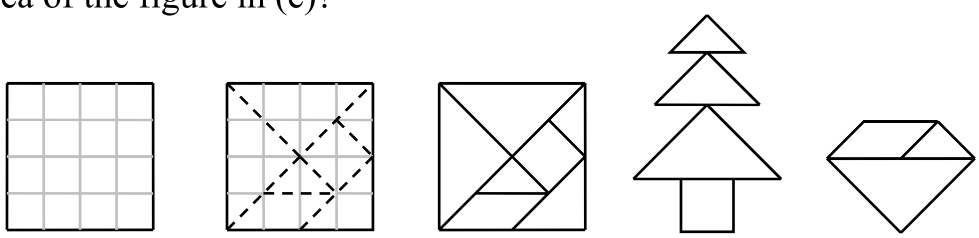
From the given pie chart, we know there are a total of 152 ÷ 19% = 800 ornaments, hence Mrs. Huang made a total of 800 × 45% = 360 ornaments on the third day.

【Suggested Solution #2】

From the given information we know that the ornaments Mrs. Huang prepared on third day are $\frac{45}{19}$ times than that of the first day. Therefore, on the third day, Mrs. Huang made a total of $152 \times \frac{45}{19} = 360$ ornaments.

Answer: B

11. The tangram puzzle is arranged as shown in the diagram. Start with a 4 by 4 grid as shown in figure (a) and cut along the dotted lines as shown in (b). This will result with seven pieces as in (c). Both figures (d) and (e) are constructed from some of these pieces. What is the ratio between the area of the figure in (d) and the area of the figure in (e)?



(a) (b) (c) (d) (e)

(A) 1 : 1 (B) 3 : 1 (C) 5 : 3 (D) 9 : 7 (E) 11 : 5

【Suggested Solution】

From diagram (a), we know the area of each grid is 1square unit, so the area of the seven-piece tangram in the diagram (b) is 1, 1, 2, 2, 2, 4, and 4 square units respectively. It follows that the area of diagram (d) is 1 + 2 + 4 + 2 = 9 square units while the area of the diagram (d) is 1 + 2 + 4 =7 square units.

Answer: D

12. The two digits of each of four consecutive two-digit numbers are multiplied, and the respective products are 24, 28, 32 and 36. What is the sum of the four consecutive numbers?

（A）120 （B）136 （C）160 （D）172 （E）190

【Suggested Solution #1】

Let us express 24 28, 32 and 36 as products of two numbers, so we have

$$24 = 3\times 8 = 4\times 6,$$
$$28 = 4\times 7,$$
$$32 = 4\times 8,$$
$$36 = 4\times 9 = 6\times 6.$$

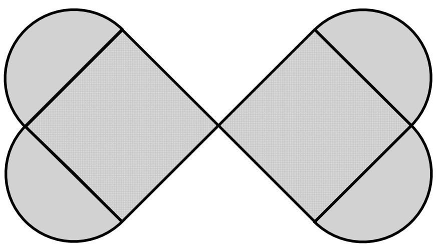
Since there are 4 consecutive numbers, then each of which is a two-digit number, and from the above series of multiplication, we know that the four consecutive numbers must be 46, 47, 48 and 49 with a sum of 190. Hence, we select option (E).

【Suggested Solution #2】

From the given information, we know the resulting product does not contain the digit 0, so the tens' place of these four consecutive numbers must be identical. Let a represent the tens' place of each of them. Since two consecutive numbers are always relatively prime, then the greatest common factor of 24, 28, 32, 36 is a. Hence, a = 4, and the ones' digit of these four consecutive numbers are 6, 7, 8 and 9, Therefore, the sum of these four consecutive numbers is 46 +47 +48 +49 = 190. We choose (E) as our answer.

Answer: E

13. Lily makes a butterfly-shaped figure as illustrated in the diagram. She uses two squares of side length 6 cm and four semicircles with diameter 6 cm. If we take π to be 3.14, what is the area of the figure?



（A）36 （B）72 （C）128.52

（D）185.04 （E）298.08

【Suggested Solution】

From the given information, the area of each square is $6^2 = 36\text{cm}^2$ while four identical semi circles will be treated as two circle, that is; the area of each circle is $\pi\left(\frac{6}{2}\right)^2 = 3.14 \times 9 = 28.26\text{cm}^2$. Thus, the area of the entire diagram is 36 ×2 + 28.26 × 2 = 128.52 cm^2.

Answer: C

14. A box contains ten black balls and ten white balls, and there are no balls outside it. In each step, Mickey is allowed to do one of the two tasks. (a) He may take a white ball and a black ball from the box and place them outside. (b) If there is at least one black ball outside the box, he may take a white ball from the box and leave it outside, and return a black ball from outside into the box. After six steps, Mickey counts the number of balls outside the box. How many possible answers are there?

(A) 2　　(B) 3　　(C) 4　　(D) 5　　(E) 6

【Suggested Solution】

Because every time one white ball is taken out from the box, so the number of balls outside the box depends on the black ball. From the rules of the game,

- when performing task (a) six times, then there will be 6 black balls outside the box;
- when performing task (a) five times and task (b) one time, there will be 4 black balls outside the box;
- when performing task (a) four times and task (b) two times, there will be 2 black balls outside the box;
- when performing task (a) three times and task (b) also three times, then no black balls will be outside the box.

We know it is not possible to perform task (b) more than three times, or else there are not enough black balls outside the box. Therefore, there could be 6, 8, 10 or 12 balls outside the box. Hence, we could select option (C).

Answer: C

15. Bucket A is one-sixth full of water while bucket B contains 60 litres of water. If we empty the water in bucket B and pour it into bucket A, then bucket A is half full of water. If instead we empty the water in bucket A into bucket B, then bucket B is full. What is the capacity, in litres, of bucket B?

(A) 70　　(B) 80　　(C) 90　　(D) 100　　(E) 180

【Suggested Solution】

From the given information, we know that $\frac{1}{3}\left(=\frac{1}{2}-\frac{1}{6}\right)$ of the capacity of the bucket is 60 litres. Hence, the capacity of the bucket A is $60\div\frac{1}{3}=180$ litres. Therefore, the capacity of bucket B is $180\times\frac{1}{2}=90$ litres.

Answer: C

16. The diagram shows an addition table of three numbers X, Y and Z. A number in the table is the sum of the two numbers, one in the same row and the other in the same column, which are represented by letters. For instance, $X+Y=16$. What is the product of the numbers X, Y and Z?

+	X	Y	Z
X	\	16	19
Y	16	\	23
Z	19	23	\

(A) 780　　(B) 800　　(C) 850

(D) 900　　(E) 960

【Suggested Solution】

From the table, we have $X + Y = 16$ and $X + Z = 19$, it follows that $Z - Y = 3$. But $Y + Z = 23$, so that $Y = 10$, $Z = 13$, $X = 6$. Hence, the product of the numbers X, Y and Z is $6\times10\times13=780$.

Answer: A

17. We define $a\,\Theta\,b=\dfrac{a+b}{2}$. If $\dfrac{3}{4}\Theta\left(\dfrac{1}{6}\Theta\square\right)=\dfrac{1}{2}$, what number is represented by $\square$?

(A) 3 (B) $\dfrac{1}{3}$ (C) $\dfrac{13}{24}$ (D) 4 (E) $\dfrac{1}{4}$

【Suggested Solution】

By observation, we know the new operation is the same as computing the average of two numbers. Next for mathematical sentence $\dfrac{3}{4}\Theta\left(\dfrac{1}{6}\Theta\square\right)=\dfrac{1}{2}$, we will first consider the bracket as another number, then the average of the number inside the brackets and $\dfrac{3}{4}$ is $\dfrac{1}{2}$, that is; $\dfrac{1}{6}\Theta\square=1-\dfrac{3}{4}=\dfrac{1}{4}$, it follows the average of $\square$ and $\dfrac{1}{6}$ equals $\dfrac{1}{4}$. Thus, we have $\square=\dfrac{1}{4}\times2-\dfrac{1}{6}=\dfrac{1}{3}$.

Answer: B

18. Mickey is asked to multiply three positive integers, but he adds them instead. Amazingly, his correct answer is equal to the correct answer for the multiplication problem. What is the sum of these three numbers?

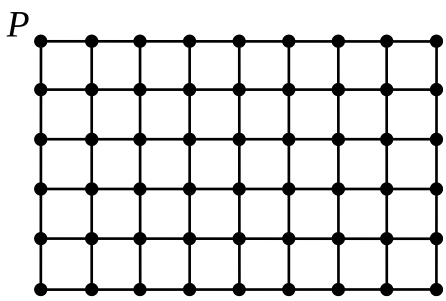
(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

【Suggested Solution】

Assume the three positive integers are arranged in increasing order as a, b and c. Obviously not all of a, b and c are equal to 1, or else $abc=1\neq1+1+1=3$, then $c\geq2$. From the given information, the possible value of a is only 1, otherwise the product of these three positive integers is more than their sum (because $abc\geq2bc=bc+bc>2b+c\geq a+b+c$), hence $1 + b + c = bc$. Similarly, when $b\geq3$, if follows that bc is greater than or equal to three times of c, but $1 + b + c$ is less than 3 times of c, this implies the value of b is 2.。Hence, $1 + 2 + c = 2c$, we have $c=3$. Therefore, sum of these three positive integers is $1 + 2 + 3 = 6$.

Answer: D

19. There are 54 grid points on the 5 by 8 grid diagram as shown, where the side of each small square is 1 cm. Starting from point P, an ant crawls from grid point to grid point along the grid lines, visiting each grid point exactly once before returning to P. What is the maximum length of its path, in cm?



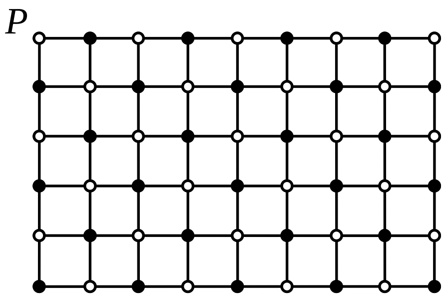
(A) 26 (B) 30 (C) 36

(D) 54 (E) 93

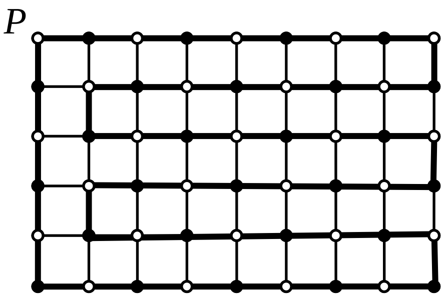
【Suggested Solution】

Let us paint each point in the given grid points alternately using black and white colors as shown in Figure (a), then there will be 27 white grid points and another 27 black grid points. Starting from the point *P*, the ant moves to black grid point first and then to white grid point, followed by another black point, then white point, ⋯, moving to alternating color grid points. The ant crawls in the path of alternating colors (via black and white) and visits all the grid points, the ant returns back to point *P*. The distance between every consecutive two adjacent grid points is 1 cm. There are a total of 54 grid points, and the start and end points are P, the circle on a total of 54 points. So the ant must also crawl in a total length of 54 cm in the complete path as illustrated in Figure (b) below. Hence, we select option (D).

Answer: D



(a)



(b)

20. When asked to find $\frac{4}{5}+\frac{5}{6}+\frac{7}{9}+\frac{9}{11}$, each of four children makes a mistake. Peter interchanges the numerator and denominator in $\frac{4}{5}$, Quentin does that to $\frac{5}{6}$, Rachel does that to $\frac{7}{9}$ and Sarah does that to $\frac{9}{11}$. Whose solution is closest to the correct answer?

(A) Peter's (B) Quentin's (C) Rachel' s

(D) Sarah' s (E) Cannot be determined

【Suggested Solution #1】

Peter is too careless and reverses the numerator and denominator of $\frac{4}{5}$, it follows the result of his calculation is $\frac{5}{4}-\frac{4}{5}=\frac{9}{20}$ more than the original sum.

Quentin interchanges the numerator and denominator of fraction $\frac{5}{6}$, then the result of his computation is $\frac{6}{5}-\frac{5}{6}=\frac{11}{30}$ bigger than the correct sum.

Rachel interchanges the numerator and denominator of $\frac{7}{9}$, this implies the results she computes as $\frac{9}{7}-\frac{7}{9}=\frac{32}{63}$ more than the original sum.

Sarah interchanges the numerator and denominator of the fraction $\frac{9}{11}$, that is; her result is $\frac{11}{9}-\frac{9}{11}=\frac{40}{99}$ greater than the original sum.

Since $\frac{11}{30}<\frac{12}{30}=\frac{2}{5}$ while $\frac{9}{20}>\frac{8}{20}=\frac{2}{5}$, $\frac{32}{63}>\frac{1}{2}>\frac{2}{5}$, $\frac{40}{99}>\frac{40}{100}=\frac{2}{5}$, so Quentin' s computation results is the one with the smallest difference with the correct value. Hence, his computation will be nearer to the correct sum.

【Suggested Solution #2】

When there are two fractions whose values are both less than 1, suppose $a>b$, then $\frac{1}{a}<\frac{1}{b}$, it follows $\frac{1}{a}-a<\frac{1}{b}-b$, so the numerator and denominator of the fractions among the four that have been computed upside down with the biggest values will have the smallest difference with the correct sum of those four fractions.

Answer: B

21. Three workers are cutting the grass in seven plots of equal size. Each worker can cut the grass in one plot in three hours. To avoid interference with each other, at most one worker may be cutting grass in the same plot at the same time. What is the required minimum number of hours before they can finish cutting the grass in all seven plots?

【Suggested Solution】

Consider the working load in cutting grass of a complete plot is 1, then the work rate of each worker in one hour is $\frac{1}{3}$. Hence, the required minimum time for 3 workers to finish cutting the grass in all seven plots is $7\div(\frac{1}{3}\times 3)=7$ hours. The following shows that the three workers can complete cutting the grasses of the 7 plots in 7 hours only. First, each worker will spend 3 hours each in cutting the grass of the plot assigned to them. Next, each of them will alternately spend 1 hour in cutting the grass of a same plot (that is the fourth plot), and finally each of them will spend 3 hours each in cutting the grass of another plot assigned to them. It means that the first worker will cut the grass in plot #4, plot #5, plot #5, plot #5; the second worker will cut the grass in plot #6, plot #4, plot #6, plot #6; while the third worker will cut the grass in plot #7, plot #7, plot #4, plot #7. Thus, it requires a total of 3 +1 +3 = 7 hours.

Answer: 007

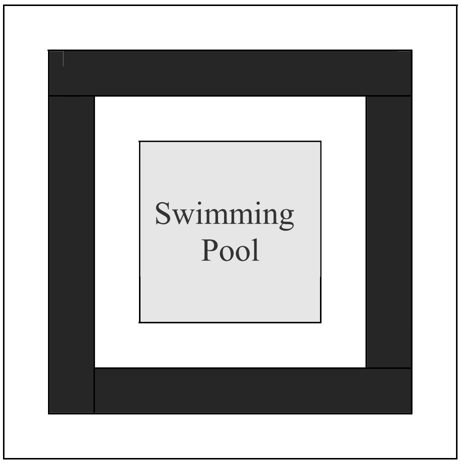
22. Jerry buys *n* books from eight bookshops. For any two bookshops, there is exactly one of the *n* books that both shops have stock. What is the value of *n*?

【Suggested Solution】

Let us consider each bookshop as one point. When two bookshops have the same title of a book, we connect these two points by one line segment. From the given information, we know there is a line segment between every two bookshops (points). Therefore, the value of *n* is $7\times 8\div 2=28$.

Answer: 028

23. The diagram shows a square swimming pool with a layer of white tiles surrounding it. This inner layer is surrounded by a layer of black tiles, and this layer is surrounded by an outer layer of white tiles. There are no gaps between layers or between the inner layers of the swimming pool. Each tile, black or white, is a square of side length 0.5 m. If the number of white tiles is 60 more than the number of black tiles, what is the area, in m^2, of the swimming pool?



【Suggested Solution #1】

Let us assume the side length of the pool as x m. From the given information, we know x must be a multiple of an integer of the length of each tile, whereas the white tiles in the innermost portion of the pool are $4\times 2x+4=8x+4$ pieces, the black tiles in the middle portion of the pool are $4(2x+2)+4=8x+12$ pieces and white tiles in the outermost portion are $4(2x+4)+4=8x+20$ pieces. Hence, we have

$$(8x+4)+(8+20)-(8x+12)=60.$$

After simplifying, we get $8x = 48$, which is equivalent as $x = 6$. Thus, the area of the pool is 36 m^2.

【Suggested Solution #2】

From the given information, we know the number of the innermost white tiles is 8 more than the black tiles, while the number of white tiles on the outermost white tiles is also 8 more than the black tiles. Hence, the white tiles on the innermost and outermost is twice the number of black tiles, it is also given that the white tiles are 60 more than the black tiles, and this implies that the number of black tiles are exactly 60 pieces. It follows that the number of white tiles is 52 pieces, so the length of the pool is $(52-4)\div 4\times 0.5=6$ m. Therefore, the area of the pool is 36 m^2.

Answer: 036

24. The number 1234567891011…198199200 is obtained by writing 1 to 200 in ascending order in a row. We now divide this number into blocks of length three, resulting in a sequence of three-digit numbers 123, 456, 789, 101, 112, and so on. What are the three numbers in the 35th block in the given sequence?

【Suggested Solution】

Since the given number sequence is divided into blocks of length three, so the number of digits involving in the first 34 blocks is $(35-1)\times 3=102$.

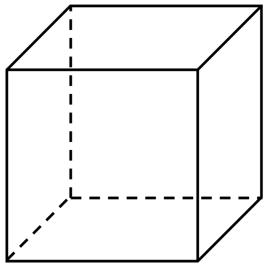
We also know that from 1 to 99, there are a total of $9+(99-10+1)\times 2=189$ digits, then the three digits belong to the 35th block is within the list of two-digit numbers. That is;

$$(102-9)\div 2=46\cdots 1.$$

Hence, the first 34 blocks of dividing the number sequence into length three include on the one-digit number from 1 to 9, two-digit numbers from 10 to 55 and also the first digit in the two-digit number 56. Therefore, the three digits in the 35th block is 657.

Answer: 657

25. The six faces of a cubical die are labeled with six different positive integers. If the numbers on any two adjacent faces differ by at least 2, what is the minimum value of the sum of these six numbers?



【Suggested Solution】

In order for the sum of all the numbers in the six surfaces to be minimum, then each of the six sides must be 1, otherwise the number appears in each face must be reduced by 1, then the sum will also be decreasing. Similarly, in a standard cube, the opposite side of a number 1 must be the number 2, or each number on each face (except the face with number 1) must decrease by 1 as well, then the sum will become smaller. We know the remaining four faces adjacent to number 2, we can predict the minimum sum will be 2 + 2 = 4, if the sum is more than 4, then each number on the four sides must each decrease by 1, so that the total will also be reduced. We can now enter in the opposite face of 4 by the number 5, then the remaining two faces which are adjacent with 5, so that the minimum sum of numbers in those two faces must be 5 +2 = 7, if more than 7, then each number on the two sides must each be decreased by 1, so that the total will also be reduced. Now we can enter in 8 in the opposite face of the number 7, at this time we have the minimum sum of those six faces, where these six numbers 1, 2, 4, 5, 7 and 8. Therefore, the sum of the six faces of the minimum number is 1 +2 +4 +5 +7 +8 = 27.

Answer: 027